Functional classes

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Structures

- signature $L = \{R_i\}_{i \in I} \cup \{f_j\}_{j \in J}$
Structures

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- structures are tuples: $A = (A, \{R_i^A\}_{i \in I}, \{f_j^A\}_{j \in J})$, $B = (B, \{R_i^B\}_{i \in I}, \{f_j^B\}_{j \in J})$ where

$$R_i^A \subseteq A^{n_i}, f_i^A : A^{m_j} \to A.$$
Structures

- **signature** $L = \{ R_i \}_{i \in I} \cup \{ f_j \}_{j \in J}$
- **structures** are tuples: $\mathcal{A} = (A, \{ R_i^A \}_{i \in I}, \{ f_j^A \}_{j \in J})$, $\mathcal{B} = (B, \{ R_i^B \}_{i \in I}, \{ f_j^B \}_{j \in J})$ where
  
  $$R_i^A \subseteq A^{n_i}, f_i^A : A^{m_j} \to A.$$ 

- **embedding** "1 – 1" map $F : \mathcal{A} \to \mathcal{B}$

  $$R_i^A(a_1, ..., a_{n_i}) \Leftrightarrow R_i^B(F(a_1), ..., F(a_{n_i})),$$
  $$F(f_i^A(a_1, ..., a_{m_j})) = f_i^B(F(a_1), ..., F(a_{m_j}))$$
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- structures are tuples: $\mathcal{A} = (A, \{R_i^A\}_{i \in I}, \{f_j^A\}_{j \in J}), $ $\mathcal{B} = (B, \{R_i^B\}_{i \in I}, \{f_j^B\}_{j \in J})$ where

$$R_i^A \subseteq A^{n_i}, f_i^A : A^{m_j} \to A.$$ 

- embedding "1–1" map $F : \mathcal{A} \to \mathcal{B}$

$$R_i^A(a_1, ..., a_{n_i}) \iff R_i^B(F(a_1), ..., F(a_{n_i})), $$

$$F(f_i^A(a_1, ..., a_{m_j})) = f_i^B(F(a_1), ..., F(a_{m_j}))$$

- embedding $\neq$ homomorphism
A \rightarrow B - there is an embedding from A into B
Notation

- \( A \hookrightarrow B \) - there is an embedding from \( A \) into \( B \)
- \( A \leq B \) - substructure, i.e. identity is an embedding
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Binomial notation

$$\binom{\mathbb{B}}{\mathbb{A}} = \{\mathbb{C} \leq \mathbb{B} : \mathbb{C} \cong \mathbb{A}\}.$$
Notation

• $\mathcal{A} \hookrightarrow \mathcal{B}$—there is an embedding from $\mathcal{A}$ into $\mathcal{B}$
• $\mathcal{A} \preceq \mathcal{B}$—substructure, i.e. identity is an embedding
• $\mathcal{A} \cong \mathcal{B}$—isomorphic structures, i.e. bijective embedding
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• Binomial notation

\[
\binom{\mathcal{B}}{\mathcal{A}} = \{ \mathcal{C} \preceq \mathcal{B} : \mathcal{C} \cong \mathcal{A} \}.
\]

• In binomial notation we do not assume that $\mathcal{A}$ is rigid
Ramsey class

Definition: A Ramsey class $K$ is a class of finite structures in signature $L = \{A, B, C\}$ such that:

- There exists a $2$-ary relation $r \in L^2$ with $r \in K$.
- For any coloring $c: (C, A) \to \{1, \ldots, r\}$, there exists a $B_0 \in K$ such that $c(B_0 A) = \text{const}$.

Then we write $C \Rightarrow (B, A)$ if $C \in K$. 

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Functional classes
Ramsey class

$\mathcal{K}$-class of finite structures in signature $L$
Ramsey class

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$A, B, C \in \mathcal{K}$ and $r \in \mathbb{N}$. 
Ramsey class

\( \mathcal{K} \)-class of finite structures in signature \( L \)

\( A, B, C \in \mathcal{K} \) and \( r \in \mathbb{N} \).

If for any coloring \( c : \binom{C}{A} \to \{1, \ldots, r\} \) there is \( B' \in \binom{C}{B} \) such that

\[
\text{const} = c(B_0) = c(B_1) = \ldots = c(B_{r-1}) = c(B'').
\]
Ramsey class

\( \mathcal{K} \)-class of finite structures in signature \( L \)
\( A, B, C \in \mathcal{K} \) and \( r \in \mathbb{N} \).

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Ramsey class

\( \mathcal{K} \)-class of finite structures in signature \( L \)

\( \mathcal{A}, \mathcal{B}, \mathcal{C} \in \mathcal{K} \) and \( r \in \mathbb{N} \).

If for any coloring \( c : \binom{\mathcal{C}}{\mathcal{A}} \to \{1, \ldots, r\} \) there is \( \mathcal{B}' \in \binom{\mathcal{C}}{\mathcal{B}} \) such that

\[
\left. c \right| \binom{\mathcal{B}'}{\mathcal{A}} = \text{const}
\]

then we write

\[
\mathcal{C} \rightarrow (\mathcal{B})^\mathcal{A}_r.
\]
Ramsey degree

Let $K$ be a class of finite structures. If for natural numbers $r$ and $t$ and structures $A$, $B$, and $C$ from $K$ we have that for every coloring $c$: $C \not\rightarrow f_1, \ldots, f_r$, there is $B_0 \subseteq A$ such that $c(B_0 A)$ takes at most $t$ many values then we write $C \rightarrow (B_0) A_{r,t}$.
Ramsey degree

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Ramsey degree

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Let $\mathcal{K}$ be a class of finite structures. If for natural numbers $r$ and $t$ and structures $A$, $B$ and $C$ from $\mathcal{K}$ we have that for every coloring

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$$C \rightarrow (B)^A_{r,t}.$$
We say that $A \in K$ has the Ramsey degree $t$ if for every $B \in K$ and every natural number $r$ there is $C \in K$ such that $C \not \rightarrow (B)_{A^r}$, and $t$ is the smallest such number.

We denote such a number by $t_K(A)$.

$A \in K$ is a Ramsey object in $K$ if $t_K(A) = 1$.

$K$ is a Ramsey class if $t_K(A) = 1$ for all $A \in K$.
We say that $A \in \mathcal{K}$ has the Ramsey degree $t$ if for every $B \in \mathcal{K}$ and every natural number $r$, there is $C \in \mathcal{K}$ such that $C \not\equiv B^r$, and $t$ is the smallest such number. We denote such a number by $t_{\mathcal{K}}(A)$. $A \in \mathcal{K}$ is a Ramsey object in $\mathcal{K}$ if $t_{\mathcal{K}}(A) = 1$. $\mathcal{K}$ is a Ramsey class if $t_{\mathcal{K}}(A) = 1$ for all $A \in \mathcal{K}$.
Ramsey objects

- We say that $A \in \mathcal{K}$ has the **Ramsey degree** $t$ if for every $B \in \mathcal{K}$ and every natural number $r$, there is $C \in \mathcal{K}$ such that $C \not \leq (B)^A_r$, and $t$ is the smallest such number.

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We say that $A \in \mathcal{K}$ has the **Ramsey degree** $t$ if for every $B \in \mathcal{K}$ and every natural number $r$ there is $C \in \mathcal{K}$ such that $C \rightarrow (B)^A_{r,t}$ and $t$ is the smallest such number.
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Functional classes
Ramsey objects

- We say that $\mathcal{A} \in \mathcal{K}$ has the **Ramsey degree** $t$ if for every $\mathcal{B} \in \mathcal{K}$ and every natural number $r$ there is $\mathcal{C} \in \mathcal{K}$ such that $\mathcal{C} \rightarrow (\mathcal{B})^{\mathcal{A}}_{r,t}$ and $t$ is the smallest such number. We denote such a number by $t_{\mathcal{K}}(\mathcal{A})$.

- $\mathcal{A} \in \mathcal{K}$ is a **Ramsey object** in $\mathcal{K}$ if $t_{\mathcal{K}}(\mathcal{A}) = 1$.

- $\mathcal{K}$ is a **Ramsey class** iff $t_{\mathcal{K}}(\mathcal{A}) = 1$ for all $\mathcal{A} \in \mathcal{K}$.
Standard

Ramsey classes of finite structures:

(Ramsey 1930) Linearly ordered sets $L = \emptyset$ or $L = \{ < \}$

(Graham, Rothschild 1971) Boolean algebras $L = \{ 0, 1, \land, \lor \}$ or $L = \{ 0, 1, \land, \lor, 0 \}$

(Graham, Leeb, Rothschild 1972) Vector spaces with lexicographical ordering $(A, +, F)$ $L = \{ 0, f_i g_i \}$ or $L = \{ 0, f_i g_i \}, <$


(Nešetřil 2007, Nguyen Van Thé 2009) Ordered metric and ultrametric spaces $(A, d, \lt)$ $L = \{ R_i \}$
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  \( L = \{0, 1, \land, \lor, '<'\} \) or \( L = \{0, 1, \land, \lor, '<', '<'\} \)

- (Graham, Leeb, Rothschild 1972) Vector spaces with lexicographical ordering \( (A, +, \mathbb{F}, \cdot) \)
  \( L = \{0, \{f_i\}_{i \in \mathbb{F}}\} \) or \( L = \{0, \{f_i\}_{i \in \mathbb{F}}, '<'\} \)
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  \[ L = \{E, <\} \]
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Even if we extend the previous list with the known examples it will be dominated by relational Ramsey classes.
Even if we extend the previous list with the known examples it will be dominated by relational Ramsey classes. So what?
Even if we extend the previous list with the known examples it will be dominated by relational Ramsey classes. So what do we know about functional side?
Ramsey classes
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- Boolean algebras
Ramsey classes

- Boolean algebras
- Vector spaces
Ramsey classes

- Boolean algebras
- Vector spaces
- *(Leeb 1973)* Trees as semilattices with convex ordering $(A, \wedge, <)$
Ramsey classes

- Boolean algebras
- Vector spaces
- (Leeb 1973) Trees as semilattices with convex ordering $(A, \wedge, <)$
- (Deuber 1975) Regular trees as semilattices with fixed branching + all terminal nodes of the same height
Ramsey objects

Ramsey objects in the class of finite Abelian groups are groups which are direct product of homocyclic groups.

Homocyclic group = product of one or more isomorphic cyclic groups.

(Ramsey objects) Ramsey objects in the class of distributive lattices are Boolean lattices. Moreover, we know degrees for all objects in this class.

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Functional classes
Ramsey objects

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- (Fouche 1999, Kechris-Sokic 2012) Ramsey objects in the class of distributive lattices are Boolean lattices. Moreover we know degrees for all objects in this class.
Jezek-Nesetril 1983

Theorem
Let $K$ be a class of finite algebras which is closed under taking products and subalgebras. Then 1-s singleton structure is a Ramsey object in this class.

Corollary
Points are Ramsey objects in the class of finite lattices. (More less all that we know about lattices).
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Sheila Oates-Williams (1988)

Theorem

Let $G$ be a group with no abelian $p$-Sylow subgroups. Let $K$ be a variety generated by $G$. Then $t_K(C_p) > 1, r_2.$

Theorem

Let $p$ be an odd prime number and let $G$ be dihedral group of order $2^p$. Let $K$ be a variety generated by $G$ and let $H_2 K$. Then

(i) If $j_{H_j} = 2^p k$ then there is $C_2 K$ such that $C_2! (H) C_2 r, r_2.

(ii) If $(j_{H_j} = 2^p k$ and $H$ has no proper direct decomposition) or $(j_{H_j} = p^k)$ then there is $C_2 K$ such that $C_2! (H) C_2 r, r_2, 1, l k.$
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Theorem

Let $G$ be a group with no abelian $p$-Sylow subgroups. Let $\mathcal{K}$ be a variety generated by $G$. Then

$$t_{\mathcal{K}}(C_p) > 1, r \geq 2.$$
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**Theorem**

Let $p$ be an odd prime number and let $G$ be dihedral group of order $2p$. Let $\mathcal{K}$ be a variety generated by $G$ and let $H \in \mathcal{K}$. Then

(i) If $|H| = 2p^k$ then there is $C \in \mathcal{K}$ such that

$$C \rightarrow (H)^{C_2}, r \geq 2.$$  

(ii) If ($|H| = 2p^k$ and $H$ has no proper direct decomposition) or ($|H| = p^k$) then there is $C \in \mathcal{K}$ such that

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(ii) If ($|H| = 2p^k$ and $H$ has no proper direct decomposition) or ($|H| = p^k$) then there is $C \in \mathcal{K}$ such that

$$C \not\to (H)^{C_2}, r > 0.$$
Relations defined in the same way.
Relations defined in the same way.

Functions are different

\[ f : r^X \rightarrow s^X; r, s \in \mathbb{N} \]

where \( r^X \) is the set of functions from \( X \) into \( \{1, ..., r\} \).
What about?

- The class of finite groups.
What about?

- The class of finite groups.
- Ramsey degrees in the class of finite abelian groups.
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- The class of finite lattices. 
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- The class of finite functions in a given arity.
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- The class of finite functions in a given arity.
- Combination of the above classes.
What about?

- The class of finite groups.
- Ramsey degrees in the class of finite abelian groups.
- The class of finite lattices.
  There is continuum many Fraïssé classes of finite lattices. (Truss-Abogatma 2014)
- The class of finite functions in a given arity.
- Combination of the above classes.
- Combination of functional and relational classes
\[ L = \{ f \} \]

- \( \mathcal{F} : (A, f^A) \), \( A \)-finite, \( f^A \)-unary function on \( A \)
\[ L = \{f\} \]

- \( \mathcal{F} : (A, f^A), \text{ } A\text{-finite, } f^A\text{-unary function on } A \)
- \( \mathcal{F}_k : (A, f^A) \in \mathcal{F}, \quad (\forall a \in A)(|\{b : f^A(b) = a\}| \leq k) \)
$L = \{ f \}$

- $\mathcal{F} : (A, f^A)$, $A$-finite, $f^A$-unary function on $A$
- $\mathcal{F}_k : (A, f^A) \in \mathcal{F}$, $(\forall a \in A) (|\{ b : f^A(b) = a \}| \leq k)$
- $\mathcal{B} : (A, f^A) \in \mathcal{F}$, $f^A$-bijection
Ordered expansions

- $\mathcal{OF}: (A, f^A, \leq^A), (A, f^A) \in \mathcal{F}$, $\leq^A \in l_0(A)$
- not good enough
Ordered expansions

- $\mathcal{OF}: (A, f^A, \leq^A), (A, f^A) \in \mathcal{F}, \leq^A \in lo(A)$
  not good enough

- $\mathcal{OF}_k: (A, f^A, \leq^A) \in \mathcal{OF}, (A, f^A) \in \mathcal{F}_k$
  no ordering is good
Ordered expansions

- \( \mathcal{OF} \): \((A, f^A, \leq^A), (A, f^A) \in \mathcal{F}, \leq^A \in \text{lo}(A) \) not good enough
- \( \mathcal{OF}_k \): \((A, f^A, \leq^A) \in \mathcal{OF}, (A, f^A) \in \mathcal{F}_k \) no ordering is good
- \( \mathcal{OB} \): \((A, f^A, \leq^A) \in \mathcal{OF}, (A, f^A) \in \mathcal{B} \)
Adjustment

- \( \mathcal{CF} : (A, f^A, \leq^A) \in \mathcal{OF}, \leq^A \text{ convex sinks, trees} \)
Adjustment

- $\mathcal{CF}: (A, f^A, \leq^A) \in \mathcal{O}\mathcal{F}, \leq^A$-convex sinks, trees
- $\mathcal{CF}_k: (A, f^A, \leq^A, (I_i^A)_i=1^n, (A, f^A) \in \mathcal{F}_k, \leq^A$-convex on sinks, $I_i^A$-on trees only ordering is not total

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Functional classes
Adjustment

- $\mathcal{CF}: (A, f^A, \leq^A) \in \mathcal{OF}$, $\leq^A$-convex on sinks, trees

- $\mathcal{CF}_k: (A, f^A, \leq^A, (I_i^A)_{i=1}^n, (A, f^A) \in \mathcal{F}_k$, $\leq^A$-convex on sinks, $I_i^A$-on trees only, ordering is not total

- $\mathcal{CB}: (A, f^A, \leq^A) \in \mathcal{CF}$, $(A, f^A) \in \mathcal{B}$ do not need ordering
The following are Ramsey classes:

\(OF, CF, CF_k, OB, CB, \mathcal{B}\).
Ramsey classes

**Theorem**

The following are Ramsey classes:

\[ \text{OF}, \text{CF}, \text{CF}_k, \text{OB}, \text{CB}, \text{B}. \]

**Fact**

There is much more Ramsey classes between \text{OF} and \text{CF} (atypical for relational classes).
There is continuum many ordered expansions of $F$ which satisfy ordering property (these are bide...able).
There is continuum many ordered expansions of $\mathcal{F}$ which satisfy ordering property (these are bidefinable).
Classes

\[ M_n\mathcal{F} : (A, f_1^A, \ldots, f_n^A), (A, f_i^A) \in \mathcal{F} \]
Classes

- $\mathcal{M}_n\mathcal{F}: (A, f_1^A, \ldots, f_n^A), (A, f_i^A) \in \mathcal{F}$
- $\mathcal{M}_n\mathcal{F}_k: (A, f_1^A, \ldots, f_n^A), (A, f_i^A) \in \mathcal{F}_k$
Classes

- $\mathcal{M}_n\mathcal{F}: (A, f^A_1, \ldots, f^A_n), (A, f^A_i) \in \mathcal{F}$
- $\mathcal{M}_n\mathcal{F}_k:(A, f^A_1, \ldots, f^A_n), (A, f^A_i) \in \mathcal{F}_k$
- $\mathcal{M}_n\mathcal{B}: (A, f^A_1, \ldots, f^A_n), (A, f^A_i) \in \mathcal{B}$
Classes

- $M_n\mathcal{F}:(A, f_1^A, \ldots, f_n^A), (A, f_i^A) \in \mathcal{F}$
- $M_n\mathcal{F}_k:(A, f_1^A, \ldots, f_n^A), (A, f_i^A) \in \mathcal{F}_k$
- $M_n\mathcal{B}:(A, f_1^A, \ldots, f_n^A), (A, f_i^A) \in \mathcal{B}$
Classes

- \( \mathcal{M}_n \mathcal{F}: (A, f_1^A, ..., f_n^A), (A, f_i^A) \in \mathcal{F} \)
- \( \mathcal{M}_n \mathcal{F}_k: (A, f_1^A, ..., f_n^A), (A, f_i^A) \in \mathcal{F}_k \)
- \( \mathcal{M}_n \mathcal{B}: (A, f_1^A, ..., f_n^A), (A, f_i^A) \in \mathcal{B} \)
Expansions

\[ \text{OM}_n\mathcal{F}(A, f^A_1, \ldots, f^A_n, \leq^A) \]

Ramsey yes but not good one


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  Ramsey yes but not good one
- $OM_n \mathcal{F}_k(A, f_1^A, ..., f_n^A, \leq^A)$
  no idea
Expansions

- $\mathcal{OM}_n F : (A, f^A_1, \ldots, f^A_n, \leq^A)$
  Ramsey yes but not good one
- $\mathcal{OM}_n F_k : (A, f^A_1, \ldots, f^A_n, \leq^A)$
  no idea
- $\mathcal{OM}_n B : (A, f^A_1, \ldots, f^A_n), (A, f^A_i) \in B$
  Ramsey yes but not good one
Expansions

- $\mathcal{OM}_n F: (A, f_1^A, ..., f_n^A, \leq^A)$
  Ramsey yes but not good one

- $\mathcal{OM}_n F_k: (A, f_1^A, ..., f_n^A, \leq^A)$
  no idea

- $\mathcal{OM}_n B: (A, f_1^A, ..., f_n^A), (A, f_i^A) \in B$
  Ramsey yes but not good one
Expansions

- $\mathcal{OM}_n F: (A, f_1^A, \ldots, f_n^A, \leq^A)$
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  no idea
- $\mathcal{OM}_n B: (A, f_1^A, \ldots, f_n^A), \ (A, f_i^A) \in B$
  Ramsey yes but not good one
Questions

• What is good subclass of $\mathcal{OM}_n\mathcal{F}$. 

$\mathcal{M}_n\mathcal{B}$ is Ramsey.
Questions

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- What to do with $M_n\mathcal{F}_k$. 

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