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ON DENSE UNIONS OF SUBSPACES

ARHANGEL'SKII, A.V

ABSTRACT: We study topological spaces that can be represented as the union of a finite or countable collection of nice dense subspaces. "Nice" here can stand for "metrizable", "Moore", "paracompact", "perfect", and so on. Such a study can be motivated by the fact that many basic examples of topological spaces have a rather simple structure: they can be represented as the union of a finite collection of metrizable subspaces. Niemytzky's plane, Mrówka's space, and Michael line are typical examples of spaces of this kind.

However, the metrizable subspaces in these examples, usually, are not all dense in the whole space. Is this feature essential? Can we construct spaces with similar combination of properties, which can be represented as the union of a finite collection of dense metrizable subspaces? Questions of this kind naturally come to mind, and they are at the origin of this investigation. The answers turn out to be not quite obvious.

Our first useful observation is that if a perfect space X is the union of a countable family of dense metrizable subspaces, then X has a uniform base. On the other hand, a paracompact space X, which is the union of two dense metrizable subspaces, need not be a p-space and need not have a base of countable order. In this talk, we present a series of results on finite dense unions of some well-structured topological spaces, and discuss some open questions in this direction.

COMPACTIFICATIONS OF A PIXLEY-ROY HYPERSPACE

ANGELO BELLA

ABSTRACT: Inspired by [1] Proposition 2.13 (No compactification of $PR(2^\omega)$ has countable tightness), we consider the general problem of embedding a Pixley-Roy hyperspace into a compact space of countable tightness.

Joint work with Masami Sakai, Kanagawa University Yokohama, Japan.

[1] A. Dow and J. Moore, Tightness in σ-compact spaces, Topology Proc., 46 (2015), 213-232.

SELECTIVE AND RAMSEY BUT NOT P-POINT

ANDREAS BLASS

Abstract: For ultrafilters on the natural numbers, Ramsey properties with various exponents are equivalent to each other and to selectivity; they imply that the ultrafilter is a P-point. I shall consider weaker Ramsey properties and a weaker version of selectivity - the weakening being just enough to allow for non-P-points. Some, but not all, of these weaker conditions are equivalent to each other. The non-equivalence part of this result can be extended to give information about wider classes of ultrafilters. There is a natural additional condition in whose presence all of the weakened selectivity and Ramsey properties become equivalent.

BASIC SCREENABLE DOWKER SPACE?

DENNIS BURKE

Abstract: We construct a basic normal screenable space Z in ZFC. Working with $\mathsf{AD}+\mathsf{DC}$ instead of Choice the space Z is easily seen to be a Dowker space. The status in ZFC is not known at this time.

COUNTING COMPACT GROUP TOPOLOGIES

W. W. COMFORT* AND DIETER REMUS

Abstract: Given a group K, let $\mathfrak{cgt}(K)$ be the set of Hausdorff compact group topologies on K. The authors ask: when $|K| = \kappa \geq \omega$, what are the possible cardinalities of a pairwise homeomorphic subset $\mathbb{V}(h) \subseteq \mathfrak{cgt}(K)$ [resp., pairwise nonhomeomorphic subset $\mathbb{V}(n) \subseteq \mathfrak{cgt}(K)$]? Revisiting (sometimes, improving) theorems of Halmos, Hulanicki, Fuchs, Hawley, Chuan/Liu and Kirku, the authors show *inter alia*:

1. Always $|\mathbb{V}(h)| \leq 2^\kappa$ and $|\mathbb{V}(n)| \leq \kappa$.

2. If K is abelian and some $\mathcal{T} \in \mathfrak{cgt}(K)$ is connected, then $|\mathbb{V}(h)| = 2^\kappa$ does occur. In particular for $\lambda \geq \omega$ and $K = \mathbb{R}^\lambda$ or $K = \mathbb{T}^\lambda$, $|\mathbb{V}(h)| = 2^{(2^\lambda)}$ does occur.

3. [K not necessarly abelian] If some $\mathcal{T} \in \mathfrak{cgt}(K)$ is connected and the connected component $Z_0(K, \mathcal{T})$ of the center of $(K, \mathcal{T})$ satisfies $\pi_1(Z_0(K, \mathcal{T})) \neq \{0\}$, then $|\mathfrak{cgt}(K)| \geq 2^{\mathfrak{c}}$.

4. Corollary to 3: Every nonsemisimple compact connected Lie group admits exactly $2^{\mathfrak{c}}$-many compact group topologies.

5. For $K = \mathbb{T}$: $|\mathbb{V}(n)| = \mathfrak{c}$ occurs in ZFC.

6. For $K = \mathbb{R}$: $|\mathbb{V}(n)| = \omega$ occurs in ZFC; $|\mathbb{V}(n)| = \omega$ is best possible in ZFC + CH; and $|\mathbb{V}(n)| > \omega$ is consistent with ZFC.

* Presented at Cornell by this co-author.

MORE ON COMPACT SPACES OF COUNTABLE TIGHTNESS

ALAN DOW

Abstract: The well-known Moore-Mrówka problem asks if every compact space of countable tightness is sequential. A set is sequentially closed if no point in the complement is the limit of a converging sequence from the set. A space is sequential if every sequentially closed set is closed. A space is sequentially compact if every sequence has a non-trivial converging subsequence. A space is C-closed if every countably compact subset is closed. We look at the old question of whether every compact C-closed space is sequential.

CH AND THE MOORE-MRÓWKA PROBLEM

TODD EISWORTH
(JOINT WORK WITH ALAN DOW)

Abstract: We will outline two proofs (both joint work with Dow) of the consistency of the Continuum Hypothesis with all compact (Hausdorff) spaces of countable tightness being sequential, thus solving the "Moore-Mrówka with CH" problem. Our techniques actually establish more than this: in particular, we prove the consistency with CH of a statement $\pi(*)$

["regular spaces of hereditarily countable π-character are C-closed"] that is strong enough to imply compact countably tight spaces are sequential in any model of $\mathfrak{c} < 2^{\mathfrak{t}}$, and whose status under PFA (or even just $\neg$CH) is unresolved.

MORE THAN HALF A LIFETIME OF DOW

K.P. HART

Abstract: I have been privileged to have had three decades of fun doing mathematics with Alan. This talk will survey the fruits of all that fun.

SEPARATING LINEAR EXPRESSIONS IN THE STONE-ČECH COMPACTIFICATION OF DIRECT SUMS

NEIL HINDMAN AND DONA STRAUSS

Abstract: A finite sequence $\vec{a} = \langle a_i \rangle_{i=1}^m$ in $\mathbb{Z} \setminus \{0\}$ is *compressed* provided $a_i \neq a_{i+1}$ for $i < m$. It is known that if $\vec{a} = \langle a_i \rangle_{i=1}^m$ and $\vec{b} = \langle b_i \rangle_{i=1}^k$ are compressed sequences in $\mathbb{Z} \setminus \{0\}$, then there exist idempotents p and q in $\beta\mathbb{Q}_d \setminus \{0\}$ such that $a_1p + a_2p + \ldots + a_mp = b_1q + b_2q + \ldots + b_kq$ if and only if $\vec{b}$ is a rational multiple of $\vec{a}$. (Here $\beta\mathbb{Q}_d$ is the Stone-Čech compactification of the set of rational numbers with the discrete topology.) We extend these results to direct sums of $\mathbb{Q}$. As a corollary, we show that if $\vec{b}$ is not a rational multiple of $\vec{a}$ and G is any torsion free commutative group, then there do not exist idempotents p and q in $\beta G_d \setminus \{0\}$ such that $a_1p + a_2p + \ldots + a_mp = b_1q + b_2q + \ldots + b_kq$.

ANTI-URYSOHN SPACES

I. JUHÁSZ

Abstract: All our spaces are Hausdorff.

Definition 0.1. 1. The space X is anti-Urysohn (in short: AU) if no two points in X have disjoint closed neighborhoods.

2. X is strongly anti-Urysohn (in short: SAU) if any two infinite closed sets in X meet AND X has more than one non-isolated points.

Theorem 0.2. *For every infinite cardinal κ there is an AU of size κ.*

We do not know if an SAU exists in ZFC, however we do have several consistent examples. For instance:

Theorem 0.3. *If $\mathfrak{r} = \mathfrak{c}$ then there is a locally countable SAU of size $\mathfrak{c}$.*

Unlike for AU's, the size of an SAU cannot be arbitrary.

Theorem 0.4. *If X is SAU then $\mathfrak{s} \leq |X| \leq 2^{2^{\mathfrak{c}}}$.*

Actually, all the examples we have are of size $\leq \mathfrak{c}$.

Clearly, a crowded SAU is AU. The existence of an SAU implies that of a crowded one:

Theorem 0.5. *If X is SAU such that for any infinite closed set F in X we have $|F| = |X|$ then we can modify its topology so that it becomes separable and crowded but it remains SAU.*

This is joint work with L. Soukup and Z. Szentmiklóssy.

ALAN AND WEAK TOPOLOGY

HEIKKI JUNNILA

ABSTRACT: I shall review my joint work with Alan and Jan Pelant on weak topology of Banach spaces, and I shall report on later developments and the status of some of the problems raised in our papers on that subject.

THE GEOMETRY OF THE BANACH SPACE $C(\mathbb{N}^*)$

PIOTR KOSZMIDER

ABSTRACT: The geometry of the Banach space $C(\mathbb{N}^*)$ of real continuous functions on $\mathbb{N}^*$ with the supremum norm is an even bigger mystery than the topology of $\mathbb{N}^*$ or the Boolean structure of the corresponding $\mathcal{P}(\mathbb{N})/Fin$. On the other hand $C(\mathbb{N}^*)$ is isometric to the classical space ℓ_∞/c_0 where the structure of both ℓ_∞ and c_0 is well understood from the Banach space theoretic point of view. We will survey the classical results, recent progress and open problems.

A FEW WORDS ON THE REAPING, I MEAN REFINEMENT, NO THAT IS THE UNSPLITTING NUMBER

CLAUDE LAFLAMME

ABSTRACT: A few words on the reaping, I mean refinement, no that is the unsplitting number.

AUTOMORPHISMS OF $\mathcal{P}(\lambda)/I_\kappa$, FOR λ UNCOUNTABLE

PAUL LARSON

ABSTRACT: We will discuss automorphisms of $\mathcal{P}(\lambda)/I_\kappa$, for $\kappa \leq \lambda$ infinite cardinals, where I_κ denotes the ideal of sets of cardinality less than κ. After surveying what we know about the state of the subject, we will present an argument which shows (among other things) that if λ is regular and at most the continuum, and the covering number for meager is greater than $\aleph_1$, then every automorphism of $\mathcal{P}(\lambda)/\mathrm{Fin}$ which is trivial on sets of cardinality less than λ is trivial.

SOME PROBLEMS ON COUNTABLE DENSE HOMOGENEOUS SPACES

JAN VAN MILL

ABSTRACT: A space X is countable dense homogeneous if for all countable dense subsets A and B of X there is a homeomorphism f of X that takes A onto B. In this talk we discuss several open problems on countable dense homogeneity and some recent progress on them.

THE ONTO MAPPING PROPERTY OF SIERPINSKI

ARNOLD W. MILLER

ABSTRACT: Define the property:

(*) There exists $\{\phi_n : \omega_1 \to \omega_1 \mid n < \omega\}$ such that for every uncountable $I \subseteq \omega_1$ there exists n such that ϕ_n maps I onto ω_1.

This is roughly what Sierpinski in his book on the continuum hypothesis refers to as P_3 but I think he brings the real number line into it. I don't know French so I cannot say for sure what he says but I think he proves that (*) follows from the continuum hypothesis. We show that

1. the existence of a Luzin set implies (*),
2. (*) implies that there exists a nonmeager set of reals of size ω_1, and
3. it is relatively consistent that (*) holds but there is no Luzin set.

All the other variant properties in this paper, (**), (S*), (S**), (B*) are shown to be equivalent to (*).

ON EMBEDDING CERTAIN PARTIAL ORDERS INTO THE P-POINTS UNDER RK AND TUKEY REDUCIBILITY

DILIP RAGHAVAN

ABSTRACT: The study of the global structure of ultrafilters on the natural numbers with respect to the quasi-orders of Rudin-Keisler and Rudin-Blass reducibility was initiated in the 1970s by Blass, Keisler, Kunen, and Rudin. In a 1973 paper Blass studied the special class of P-points under the quasi-ordering of Rudin-Keisler reducibility. He asked what partially ordered sets can be embedded into the P-points when the P-points are equipped with this ordering. This question is of most interest under some hypothesis that guarantees the existence of many P-points, such as Martin's axiom for σ-centered posets. In his 1973 paper he showed under this assumption that both ω_1 and the reals can be embedded. This result was later repeated for the coarser notion of Tukey reducibility. We prove in this paper that Martin's axiom for σ-centered posets implies that every partial order of size at most continuum can be embedded into the P-points both under Rudin-Keisler and Tukey reducibility.

AUTOMORPHISMS OF LARGE ALGEBRAS MODULO FINITE

JURIS STEPRANS

ABSTRACT: The question of constructing non-trivial automorphisms of algebras of the form $\mathcal{P}(\kappa)/[\kappa]^{<\aleph_0}$ will be examined and some cases where this is not possible identified.

WIJSMAN HYPERSPACES OF NON-SEPARABLE METRIC SPACES

PAUL SZEPTYCKI AND RODRIGO HERNÁNDEZ-GUTIÉRREZ

ABSTRACT: Recent advances on normality of the hyperspace of nonseparable metric space will be presented.

SOKOLOV PROPERTY IN LINDELÖF P-SPACES

VLADIMIR V. TKACHUK

ABSTRACT: We will show that for every Lindelöf P-space a weaker version of the Sokolov property holds. Besides, if K is a scattered Eberlein compact space and X is obtained from K by declaring open all G_δ-subsets of K, then X is monotonically Sokolov. The proof of this statement uses the fact that every Lindelöf subspace of a scattered Eberlein compact space must be σ-compact; this result seems to be interesting in itself. We also give an example of a Lindelöf P-space X such that $C_p(X)$ has uncountable extent. In particular, neither X nor $C_p(X)$ has the Sokolov property.

RAMSEY-THEORETIC ANALYSIS OF FINITE-DIMENSIONAL ℓ_P'S

STEVO TODORCEVIC

ABSTRACT: This will be an overview of results about linear isometries between finite dimensional ℓ_p spaces from a Ramsey-theoretic viewpoint. We shall try to connect this with standard Ramsey theory via a new notion of spread.

This is a part of a joint work with V. Ferenczi and L. Lopez-Abad.

MY JOINT WORK WITH ALAN DOW ON ψ-SPACES, 2010-2012

JERRY E. VAUGHAN

Abstract: We discussion our recent joint work with Alan Dow on ψ-spaces and suggest directions for further research.

THE SEARCH FOR UNCOUNTABLE JAKOVLEV SPACES

WILLIAM WEISS

Abstract: Jakovlev spaces are a relatively simple type of locally compact, first countable scattered space, named for the person who first constructed an uncountable one using the continuum hypothesis. Other examples have since been constructed, but we have not yet been able to make do without some extra set-theoretic hypotheses.

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