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Tutorials

Tutorials

Computable Structure Theory
Computability Theory

Antonio Montalbán
University of California, Berkeley, USA

We will introduce the basic concepts of Computable Structure Theory, give a taste of a few of the methods used, and end by explaining a few more recent results. Computable structure theory studies mathematical structures from a computational viewpoint. Our goal is to find interactions between computational properties and algebraic properties of structures, and also to understand to what extent those interactions are possible. Here are two basic questions we look at: How difficult is it to find a representation of a give structure? How difficult is it to find isomorphisms between structures? We will end mentioning some recent work relating the famous Vaught's conjecture from model theory to computability theoretic properties. (By computability here I mostly mean between non-computable objects, as in computability theory, and I'm not referring to time- or space-complexity as in computer science.)

Dualities
Non-classical Logics.

Luca Spada,
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Categorical dualities are pivotal tools in mathematical logic as well as in several other branches of mathematics. In logic, they are often used to give concrete semantics (usually based on geometric objects) to abstract syntax (usually encoded in algebraic terms). Standard examples are Stone duality for Boolean algebras, Priestley duality for distributive lattices, the duality between finitely presented MV-algebras and rational polyhedra, and Gelfand duality for C*-algebras. The scope of the tutorial is to cover all these fundamental dualities with a unified approach stemming from the classical Galois connection between ideals of polynomials and zero-sets of polynomial functions. This will provide a general method that, given a class of structures, constructs a dual adjunction between the latter and a class of objects that can be naturally topologised. Thanks to this approach unexpected connections with Hilberts Nullstellensatz, Birkhoff subdirect representation, and Gel'fand transform will emerge during the tutorial.

Continuous model theory and applications
Model Theory

Isaac Goldbring
University of Illinois at Chicago, USA.

Continuous logic is a relatively recent logic, introduced by I. Ben Yaacov and A. Usvyatsov and further developed by I. Ben Yaacov, A. Berenstein, C.W. Henson, and A. Usvyatsov, suited for studying structures from analysis that are based on metric spaces. The main novelty in this logic is that formulae are no longer true or false but can take on any value in a bounded interval of the real numbers. The model theory associated to this logic resembles classical model theory in that many of the familiar theorems (e.g. the compactness theorem, the omitting types theorem, the Beth Definability theorem) have continuous analogues that are, in fact, generalizations of their "discrete" counterparts.

In this tutorial, I will present the basics of continuous logic, defining the syntax and semantics of this logic as well as presenting some examples of continuous theories. I will also present the continuous ultraproduct construction and show how this yields the compactness theorem for continuous logic. I will then move on to two recent applications of continuous model theory. I will first discuss a result of I. Farah, B. Hart, and D. Sherman, where they show how to use model-theoretic (in)stability to answer questions about isomorphic ultrapowers asked by many people in the operator algebra community. Then I will discuss how I. Farah and S. Shelah use saturation of certain continuous reduced products to show, assuming the Continuum Hypothesis, that the Stone-Cech remainder of the real line has many nontrivial autohomeomorphisms.

Compactness and Reflection properties as a motivation for set theoretical Investigations.

Set Theory

Menachem Magidor
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Reflection principles come out naturally in many mathematical contexts in which one studies uncountable structures. Examples were studied in Topology, Algebra, Infinite combinatorics etc. Many times these principles and problems associated with them are important source for interesting Set Theoretical investigations.

A reflection principle for a property of mathematical structures of certain kind is the statement that a structure having the property has a *small* substructure with the property. *"Small"* typically means *"having smaller cardinality"* but there are other notions of *"smallness"*.

Few examples:

1. Suppose that a collection of sets does not have a one to one choice function. Does there exists a smaller cardinality subfamily with the same property?
2. Suppose that a topological space X is not metric. (In order to avoid trivialities assume that X satisfies some minimal conditions necessary for being metric. e.g. It is fist countable.) Does the space has a smaller subspace which is not metric?
3. Suppose that the Abelian group G is not free. Does it have a smaller subgroup which is not free?
4. Stationary reflection: Let κ be a regular cardinal and $S \subseteq \kappa$ is stationary. Does S reflect? Namely does there exists an ordinal $\beta < \kappa$ such that $S \cap \beta$ is stationary in β?

Compactness properties are dual properties to reflection properties. Typically they claim that if for a given structure , we have many small substructure with the property (e.g. for very smaller cardinality sub- structure), then the whole structure has this property. A reflection principle is equivalent to a compactness property for the negation of the property and vice versa. Few examples:

- The tree property for a cardinal κ can be phrased as compactness property. Namely we have a κ tree (A tree with κ levels, each level of size less than κ.) We know that every proper initial segment of the tree has a cofinal branch. The tree property is the statement that the whole tree has a cofinal branch.
- G is an Abelian group such that every smaller cardinality subgroup can be embedded into a direct product of copies of the integers: Z. Can G be so embedded?
- X is a topological space such that it is λ collection- wise Hausdorff. Namely very discrete closed set of cardinality $\leq \lambda$ can be separated by a disjoint collection of open sets. Under what conditions can we infer that X is fully collection- wise Hausdorff. (Namely every discrete closed subset can be separated)?

Problems of this type are closely connected with the basic properties of the universe of Set Theory like the existence of large cardinals or combinatorial principles like $\square_\kappa$. There are interesting connections between different properties. Many times they give a natural motivation for additional axioms for Set Theory. In the tutorial we shall give few examples of such properties and their set theoretical connections. We are going to assume some familiarity with basic set theoretical concepts, but they will be rather elementary. For instance we shall give the definition of the large cardinals we shall use. Forcing constructions will be used as black box.

Plenary talks

Plenary talks

On the semantics of non-commutative geometry and quantum mechanics

Boris Zilber
University of Oxford, England.

The well-known duality of classical algebraic geometry between affine varieties and their co-ordinate rings has a perfect analogue in the theory of commutative C^*-algebras, which can be seen by the Gel'fand-Naimark theorem as the algebras of continuous complex-valued functions on a compact Hausdorff space. We interpret this as the Syntax-Semantics duality. In modern geometry and physics one deals with much more advanced generalisations of co-ordinate algebras, such as schemes, stacks and non-commutative C^*-algebras, where a geometric counterpart is no longer readily available and in many cases is believed impossible.

I will discuss some results of a model-theoretic project which challenges this point of view and has applications in quantum mechanics.

On Normal Numbers

Verónica Becher
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Normality is a basic form of randomness. A real number is normal to a given base if each block of digits of equal length occurs in the expansion of the number with the same limit frequency. This definition was introduced by mile Borel more than one hundred years ago, but not much is known about normal numbers. One of the famous open problems is whether the usual mathematical constants, as p, e, and v2, are normal to any base. In this talk I will summarize recent results on normal numbers, including the current known examples of normal numbers and I will actually exhibit the expansion of a number normal to every base, that we computed at the University of Buenos Aires.

Positive provability logic and reflection principles in arithmetic

Lev Beklemishev
Steklov Mathematical Institute, Moscow, Russia.

We deal with the fragment of modal logic consisting of implications of formulas built up from the variables and the constant 'true' by conjunction and diamonds only. The weaker language allows one to interpret the diamonds as the uniform reflection schemata in arithmetic, possibly of unrestricted logical complexity. We formulate an arithmetically complete calculus with modalities labeled by natural numbers and ω, where ω corresponds to the full uniform reflection schema, whereas $n < \omega$ corresponds to its restriction to arithmetical Π_{n+1}-formulas. This calculus is shown to be complete w.r.t. a suitable class of finite Kripke models and to be decidable in polynomial time.

A theory of descriptions revisited

Oswaldo Chateaubriand
Pontificia Universidade Católica de Rio de Janeiro, Brasil.

Many years ago I developed a theory of descriptions that combines features of Frege's theory with features that form part of Russell's theory. I presented it in various talks, in my book Logical Forms, and in several later publications. Although it was well received by audiences-including by Quine and by Strawson at a talk at the International Congress of Philosophy in Boston 1998-it has not had an impact in the literature. Since I never presented it to a congress of logicians I would like to have another go at it and see the reactions. My claim on its behalf is that it is a better theory than any others I know, and that for indefinitely many sentences it gives a correct account of their truth conditions where other theories do not. The main references are the following:

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Reducts and expansions of residuated lattices

Peter Jipsen
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Algebraic logic considers logical calculi from an algebraic view point, allowing techniques from algebra and category theory to be applied to logic and vice versa. A residuated lattice $(A, \wedge, \vee, \cdot, 1, \backslash, /)$ is a combination of a lattice $(A, \wedge, \vee)$, a monoid $(A, \cdot, 1)$ and residuals $\backslash, /$ that satisfy $xy \leq z \iff y \leq x\backslash z \iff x \leq z/y$ for all $x, y, z \in A$. The class of all such algebras provides algebraic semantics of substructural logics, where the lattice meet and join are conjunction and disjunction, while the monoid and residuals are a dynamic conjunction (fusion) with its associated implications. Other nonclassical propositional logics that have fewer or more logical connectives are obtained by removing some of the operations of residuated lattices (reducts) or adding new operations (expansions).

In this talk we consider residuated semirings, modal residuated lattices, FL'-algebras and Heyting residuated lattices. The latter classes are generalizations of modal algebras and relation algebras, hence they connect nonclassical algebraic logic with the original classical version. We indicate how frame semantics of residuated lattices can be adapted to these more general settings, giving dualities to categories of contexts expanded with relations, thus linking residuated lattices with formal concept analysis. Applications to amalgamation, interpolation and algebraic proof theory are examined. We also show how automated theorem provers and model finders can be used to prove results about the (strong) amalgamation property.

Towards a dual Ramsey theory of trees

Stevo Todorcevic
University of Toronto, Canada.

The classical Ramsey theory of trees that is based on the notion of strong sub-tree and the Halpern-Lauchli theorem as the corresponding pigeonhole principle has seen over the last fifty years many applications. There is a natural problem that asks for the development of the corresponding dual theory. In this lecture we survey recent progress in that direction and list the corresponding applications. In particular, we shall cover some recent joint work with A. Aviles and K. Tyros.

Computability theory

Computability theory

Invited Talks.

Digital morphogenesis via Schelling segregation.

Andy Lewis
London School of Economics, UK

Schelling's model of segregation looks to explain the way in which particles or agents of two types in an initially random configuration may come to arrange themselves spatially into configurations consisting of large homogeneous clusters, i.e. connected regions consisting of only one type. As one of the earliest agent based models studied by economists and perhaps the most famous model of self-organising behaviour, it also has direct links to areas at the interface between computer science and statistical mechanics, such as the Ising model, Hopfield networks and the study of contagion and cascading phenomena in networks.

While the model has been extensively studied it has largely resisted rigorous analysis, prior results from the literature generally pertaining to variants of the model which are tweaked so as to be amenable to standard techniques from statistical mechanics or stochastic evolutionary game theory. In 2012 Brandt, Immorlica, Kamath and Kleinberg provided the first rigorous analysis of the unperturbed model, for a specific set of input parameters. We have now provided a rigorous analysis of the model's behaviour much more generally and have established some surprising forms of threshold behaviour, notably the existence of situations where an *increased* level of intolerance for neighbouring agents of opposite type leads almost certainly to *decreased* segregation.

Joint work with George Barmpalias and Richard Elwes.

Monadic Second Order Logic on Linear orderings.

Olivier Carton
LIAFA, CNRS - Universit Paris Diderot, France

In this talk, we will review some past and recent results on the MSO logic on Linear orderings. This topic can be seen as a natural continuation of a long line of research that aims at devising robust and decidable classes of languages with equivalent presentations based on logics; automata, and algebras.

The central objects of the talk are the so-called countable words, i.e., totally ordered countable sets equipped with functions mapping positions to letters in some finite alphabet. Like in the classical case of finite and omega-words, some languages of countable words can be described by means of Monadic Second-Order (MSO) logic, which makes use of quantifications over single positions as well as quantifications over sets of positions.

After a summary of some classical results concerning the MSO theories of linear orders, I will present an algebraic model for recognizing languages of countable words and show that this notion of recognizability is effectively equivalent to definability in MSO logic. This gives an alternative proof of the decidability of the MSO theory of the ordered rational numbers. It also implies the first known collapse result for MSO over countable linear orderings.

The talk is based on a joint work with Thomas Colcombet and Gabriele Puppis.

Contributed Talks.

Implications of computer science principles in quantum physics

Gabriel Senno, Santiago Figueira, Ariel Bendersky, Gonzalo de La Torre and Antonio Acín
UBA, Argentina and Spain.

We present an ongoing research program on the implications of some principles from computer science in the foundations of quantum mechanics. In this talk we will comment on three results we have so far:

- **Distinguishability of different classical mixtures of quantum states (Acn, Bendersky, de La Torre, Figueira, Senno).** A quantum system is said to be in a pure state when we have maximal knowledge on the state of the system. Mixed states, on the other hand, describe ensembles of pure states of which we have classical uncertainty, like in a macrostate appearing in statistical mechanics. These are called proper mixed states.

  It is a fundamental theorem of quantum theory that, given a (proper) mixed state, there are no means to find out which of the compatible, and sometimes physically very different, ensembles were used at the preparation stage.

  However, we prove that, if the mixture is done algorithmically, the preparations can be distinguished with arbitrarily high probability.
- **Computability loophole for Bell's inequalities (Acn, Bendersky, de La Torre, Figueira, Senno).** Non-locality is one of the most intriguing features of quantum mechanics. One of the first to notice such a bizarre feature were Einstein, Podolsky and Rosen (1935) who claimed that quantum mechanics had to be wrong because it allowed for some action at a distance: the outcomes of an experiment performed in one location affected instantaneously the outcomes of another experiment performed at a distance. In 1964 J. S. Bell proposed a way to test if nature behaved in a local or non-local way. His setup involved the measurement of correlations between two separated setups, each setup had a choice to measure one of several observables, and at the end of the day, bringing together the results from both setups and analysing their correlations, it was possible to tell if those came from a local or non-local bipartite probability distribution.

  We prove that if the measurements choices are made following an algorithm —or, equivalently, because of the Church-Turing thesis, following any classical mechanical procedure—, then non-locality can be deterministically simulated, providing the aforementioned loophole.
- **Uncomputability of the sequence of outputs of an infinite repetition of a GHZ experiment (Figueira, Senno).** A Greenberger-Horne-Zeilinger(GHZ) experiment is similar to a test of Bell's inequalities, except that is uses three or more entangled particles, rather than two. With specific settings of GHZ experiments, it is possible to demonstrate absolute contradictions between the predictions of local hidden variable theories and those of quantum mechanics, whereas tests of Bell's inequalities only demonstrate contradictions of a statistical nature.

  We show how, under some physical and philosophical assumptions, the sequence of outputs of an infinite repetition of a certain GHZ experiment turns out to be uncomputable.

Decidability of Gödel Modal Logics KD45, KT45 and their extensions.

Ricardo Oscar Rodriguez
Departamento de Computacin. FCEyN-UBA, Argentina.

Gödel modal logics combine Kripke frames of modal logics with the semantics of the well-known fuzzy Gödel logic. These logics, symbolized by GK, have been investigated in some detail by Caicedo and Rodríguez [4, 3] and Metcalfe and Olivetti [8, 9]. More general approaches, focussing mainly on finite-valued modal logics, have been developed by Fitting [7], Priest [10], and Bou et al. [1].

Axiomatizations were obtained for the box and diamond fragments of GK in [4]. It was subsequently shown in [3] that the full logic GK is axiomatized either by adding the Fischer Servi axioms for intuitionistic modal logic IK (see [6]) to the union of the axioms for both fragments, or by adding the prelinearity axiom for Gödel logic to IK. Decidability of the diamond fragment of GK was established in [4], using the fact that the fragment has the finite model property with respect to its Kripke semantics. This finite model property fails for the box fragment of GK, but decidability and PSPACE-completeness for this fragment was established in [8, 9] using analytic Gentzen-style proof systems. In the general case of the full logic GK, the finite model property also fails with respect to its Kripke semantics, and, hence, it is not possible to prove decidability using this argument.

Taking this observation into account, in [5], the authors were able to come up with a slightly different, but equivalent, semantic characterization of the logic GK, for which the finite model property holds. This characterization, call it GFK, is defined by extending a GK model by a function T that assigns to each world a finite set of truth values in $[0;1]$ and restricting the valuation of box- and diamond-formulae (but nothing else) to this set. They then are able to prove the decidability of validity in full GK. Unfortunately, a similar strategy to establish decidability for extension of GK is impossible due to technical problems.

In a forthcoming paper [2], the authors show the logics GKD45, GKT45 and their extensions admitting a more simplified, but equivalent, semantics instead of the Kripke one. The simplified semantics is given by a structure $M = (W, e, \pi)$ where W is a set of worlds, $e : W \times Var \to [0, 1]$ is an evaluation of propositional variables in each world, and $\pi : W \to [0, 1]$ is now a fuzzy set of worlds instead of a fuzzy relation as in the Kripke semantics. Then, the truth value in each world w of formulae is recursively defined as usual with the exception of the following rules for $\Box$ and $\Diamond$:

- $e(\Box\varphi, w) = \inf_{w' \in W}\{\pi(w') \Rightarrow_G e(\varphi, w')\}$
- $e(\Diamond\varphi, w) = \sup_{w' \in W}\{\min(\pi(w'), e(\varphi, w'))\}$

Note that if Φ is a propositional combination of modal formulae, then its truth value does not depend on the particular world. In this alternative semantics, we are able to overcome the technical problems mentioned above and to prove decidability of validity for their logics using the strategy proposes in [5]. This is the contribution of this paper.

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First Order Hypothetical Logic of Proofs

Gabriela Steren and Eduardo Bonelli
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The Logic of Proofs (LP) is a refinement of modal logic introduced by Artemov in 1995, in which the modality []A is revisited as [t]A, where t is an expression that bears witness to the validity of A. It enjoys arithmetical soundness and completeness and is capable of reflecting its own proofs ($\vdash$ A implies $\vdash$ [t]A, for some t). This suggests that computational interpretations in terms of the Curry-Howard isomorphism could yield programming languages supporting a uniform treatment of programs and type derivations. Some avenues in this direction have been explored (eg. certifying mobile computation and history-aware computation) and, more recently, the Hypothetical Logic of Proofs (HLP) which captures the full, classical propositional LP.

Extending the previous work to first order logic has presented a new challenge. In 2011, Artemov and Yavorskaya introduced the First Order Logic of Proofs (FOLP), capable of realizing first-order modal logic S4 and, therefore, the first-order intuitionistic logic HPC. FOLP enjoys a natural provability interpretation; this provides a semantics of explicit proofs for first-order S4 and HPC compliant with Brouwer-Heyting-Kolmogorov requirements. FOLP opened the door to a general theory of first-order justification.

In the current work in progress, we define the First Order Hypothetical Logic of Proofs (FOHLP), a natural deduction system which bears the same relation to FOLP as HLP does to propositional LP. We show that FOHLP can prove all FOLP-theorems, and we provide a translation from FOHLP to FOLP which preserves derivability.

Definability Results for Some Fragments of XPath with Equality Tests

Sergio Abriola, Santiago Figueira and Mara Emilia Descotte
Universidad de Buenos Aires.

An XML document is a data tree, i.e. a tree whose every node contains a tag or label (such as *LastName*) from a finite domain, and a data value (such as *Smith*) from an infinite domain. XPath has syntactic operators to navigate the tree using the 'child', 'parent', 'sibling', etc. accessibility relations, and can make tests on intermediate nodes. Core-XPath [3] is the fragment of XPath 1.0 containing only the navigational behavior of XPath. It can express properties of the underlying tree structure of the XML document, such as "*the root of the tree has a child labeled a and a child labeled b*", but it cannot express conditions on the actual data contained in the attributes, such as "*the root of the tree has two children with same tag a but different data value*". However, Core-Data-XPath [1], here called $\text{XPath}_=$, can. Indeed, $\text{XPath}_=$ is the extension of Core-XPath with (in)equality tests between attributes of elements in an XML document.

In a recent paper [2], the expressive power of $\text{XPath}_=$ was studied, from a logical and modal model theoretical point of view. A notion of bisimulation is introduced for some fragments of $\text{XPath}_=$, and a van Benthem like characterization theorem is shown for some of them. In this work we show a definability theorem, which answers the basic question of when a class of data trees is definable by a set of formulas, or by a single formula, over two fragments of $\text{XPath}_=$: the *downward* fragment (which only has the 'child' accessibility relation) and the *vertical* fragment (which has both 'child' and 'parent' axes).

Our main result is the analog of the classic first-order definability theorem, which can be stated as follows:

> *A class of models K is definable by means of a set of first-order formulas if and only if K is closed under ultraproducts and isomorphisms, and the complement of K is closed under ultrapowers. Also K is definable by a single first-order formula if and only if both K and its complement are closed under ultraproducts and isomorphisms.*

The above result was adapted to the context of many modal logics, where the notion of *isomorphism* is replaced by the weaker concept of *bisimulation* (the one which turns to be adequate for the chosen modal logic). Thus definability theorems were established for many modal logics.

Our definability theorems for $\text{XPath}_=$ themselves are shown using rather known techniques. The main contribution, however, is to devise and calibrate the adequate notions to be used in the $\text{XPath}_=$ scenario, and to study the subtle interaction between them:

- *Bisimulation*: already introduced in [2], it is the counterpart of *isomorphisms* in the classical theorem for first-order logic. In [2] it is shown that if two (possibly infinite) data trees are bisimilar then they are logically equivalent (that is, they are not distinguishable by an $\text{XPath}_=$ formula) but that the converse is not true in general.

- *Saturation*: we define and study the new notion of *XPath$_=$-saturation*. We show that for XPath$_=$-saturated data trees being bisimilar is the same as being logically equivalent. It is also shown that a 2-saturated data tree (regarded as a first-order structure) is already XPath$_=$-saturated.
- *Utlraproducts*: contrary to other adaptations of the classical first-order definability theorem to modal logics, in our case we have to adjust also the notion of *ultraproduct*, and so we work with a variant of it called *quasi-ultraproduct*. The reason is that we must not abandon the universe of data trees, as these are the only allowed models of XPath$_=$.

There are many works in the literature studying the expressive power of Core-XPath, but all these consider the navigational fragment of XPath. A first step towards the study of the expressive power of XPath when equipped with (in)equality test over data trees, is the recent paper [2]. We aim to shed more light in this direction.

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Compressibility of normality: an overview of recent results

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This overview deals with the characterization of normality by compressibility using finite automata. A fundamental theorem relates normality and finite automata: an infinite word is normal to a given alphabet if and only if it cannot be compressed by lossless finite transducers. These are deterministic finite automata with injective input-output behavior. This result was first obtained by joining a theorem by Schnorr and Stimm [12] with a theorem by Dai, Lathrop, Lutz and Mayordomo [6]. A self-contained proof can be found in [4].

What is the true power needed to compress normal words? Of course, transducers augmented with enough computational power are equivalent to a Turing machine, hence they can compress computable normal infinite words. Here we analyze different ways of augmenting a plain deterministic transducer. We consider deterministic versus real-time non-deterministic and non-real-time transition functions; having zero, one, or more counters; with or without a stack and with or without the ability to read the input more than once.

The power gained by allowing non-deterministic behavior is not obvious and depends on the context. Deterministic and the non-deterministic automata recognize the same sets of finite words, known as the rational sets of finite words. However, deterministic Büchi automata are strictly less expressive than the non-deterministic ones (see [10]). For instance, deterministic Büchi automata recognize a proper subclass of sets of infinite words than the non-deterministic ones. Furthermore, functions and relations realized by deterministic transducers are proper subclasses of rational functions and relations realized by non-deterministic ones.

We prove that lossless transducers, even non-deterministic non-real-time ones, but with no extra memory or just a single counter, cannot compress normal infinite words. Adding memory yields compressibility results: non-real-time non-deterministic transducers with more than one counter can compress some normal words. Also real-time non-deterministic transducers with a stack can do it.

We also consider the power of compression of two-way transducers. As opposed to the one-way machines mentioned above, two-way transducers have an input reading head that may be moved backwards to re-read parts of the input several times.

The introduction of two-way transducers can be traced back to the very beginning of the study of transducers [1]. Two-way transducers have a nice logical characterization [7] and the equivalence of two of them is a decidable problem [8, 9]. The interest in these transducers has been recently renewed by the introduction of an equivalent model called streaming string transducers [2].

The definition of compressibility used for one-way machines does not generalize in a unique way to two-way machines. A significant part of the work we did is considering several ways in which the definition may be generalized and prove that they are all equivalent. The extent of this equivalency is stronger for deterministic two-way transducer than for non-deterministic ones. This implies that results for non-deterministic two-way automata are not strict generalizations of the results for the deterministic case.

	extra memory	deterministic	non-deterministic	non-real-time
one-way	none	Not compress	Not compress	Not compress
	one counter	Not compress	Not compress	Not compress
	multiple counters	Not compress	Not compress	Compress*
	one stack	?	Compress	Compress
	one stack and one counter	Compress	Compress	Compress*
two-way	none	Not compress	Not compress	Not compress
	one counter	Collapses	Collapses	Collapses

Table 1: Compressibility of normal infinite words by different kinds of transducers.

The main result on two-way automata is that normal infinite words are not compressible by deterministic nor by non-deterministic two-way transducers. However, there exist examples which show the traditional way of defining compressibility collapses for two-way transducers with unbounded memory, even in its simplest form of a single counter.

Table 1 summarizes the results about compressibility of normal infinite words by different kinds of transducers. The right-most three columns represent different levels of restrictions on the transitions. The first column represents determinism, that is, there is exactly one transition that leaves a given state by reading a given symbol. The second column represents the possibility that several transitions leaving the same state read the same symbol. The restriction represented in the third column adds the possibility of also having transitions that do not read any symbol (usually called λ-transitions). The rows of the table represent different combinations of a reading model and a memory model. In all cases there is a bounded memory represented by states, and each row details possible additions of counters or stacks. The realized relation is assumed to be bounded-to-one. The case of a deterministic transducers with a single stack remains open. Turing complete models are marked with an asterisk.

The results in this overview are fully developed in [11] and included in our papers [3] and [5].

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Normality in non-integer bases

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One of the weakest ways in which an infinite sequence of elements in a finite alphabet Σ may be called "random" is that of normality, which was introduced by Borel in 1909 and which may be regarded as a "law of large numbers" for blocks of events.

Definition 1. *Let b be an integer base and Σ be the alphabet in b digits. An infinite sequence $S \in \Sigma^\infty$ is called normal if, for all lengths $k \in \mathbb{N}$ and all $\alpha \in \Sigma^k$,*

$$\lim_{N\to\infty} \frac{1}{N1} \sum_{i=0}^{N} \delta_\alpha(S_{i1}, \ldots, S_{ik}) = |b|^{-k} \tag{1}$$

where $\delta_\alpha(\beta)$ is the Dirac delta function.

A real number x is called *normal in base b* if its expansion in base b is normal, and it is called *absolutely normal* if it is normal in all integer bases.

A result by Schnorr and Stimm [1] establishes that x is normal in base b if and only if no martingale whose betting factors are computed by a deterministic finite automaton (DFA) succeeds on the expansion of x in base b.[1] We say that x is polynomial-time random in base b if no polynomial-time martingale in the alphabet of b digits succeeds on the expansion of x in base b. The result of [1] hence implies that polynomial-time random in base b is stronger than normality in base b. It was recently shown [3] that polynomial-time randomness is base invariant, so that, by the preceding remark, being polynomial-time random in any base implies being normal for all bases, i.e. being absolutely normal. The converse is not true, since there are absolutely normal numbers which are computable in polynomial time.

An extension of these notions to non-integer bases is made possible once one makes the observation that x is normal in base b if and only if $xb^n \mod 1$ is a sequence uniformly distributed (u.d.) on the unit interval. Thus, a natural question that was asked in [3] is the following:

Question. If x is polynomial-time random, is it true that $(x\beta^n)_{n\in\mathbb{N}}$ is u.d. modulo 1 for all rational β?

The distribution of $(x\beta^n)_n \mod 1$ for rational β is, however, fairly intractable. It is unknown, for instance, if $((3/2)^n)_n$ is u.d. modulo 1.

Yet, there is a class of non-integer reals for which the question may be readily handled.

Definition 2. *A real number β is a Pisot number if $|\beta| > 1$ and β is the root of a monic polynomial in $\mathbb{Z}[X]$ such that $|z| < 1$ for all the other roots z of $p_\beta(x) \in \mathbb{Z}[X]$, the monic polynomial that has β as a root of smallest degree.*

The formulation of our Question in terms of mod 1 equidistribution allows us to understand normality as what ergodic theory calls *genericity*, an equivalence which boils down to two facts: 1) the map $T_\beta(x) := (\beta x)$ mod 1 on $[0, 1)$ is equivalent to a "shift" rightwards in the space of sequences $\{0, \ldots, \lfloor\beta\rfloor - 1\}^{\mathbb{N}}$ when x is mapped to its base β expansion; 2) $(x\beta^n) \mod 1 = T_\beta^n(x)$.

Yet, when a noninteger base β is considered, 2) is immediately false, while 1) has no clear reformulation, since there is no obvious candidate for a space of sequences that "represent" numbers in base β. It is here that the theory of β-shifts and β-representations, developed, among others, by Parry [4] and Bertrand [2], helps fill in the missing pieces.

Once the space of sequences that represent numbers in base β is defined, it is equipped with a natural shift transformation and a measure P_β called the *Parry measure*, which plays the same role that the uniform or Lebesgue measure played in integer representation. Indeed, a useful result by Bertrand says that, when β is Pisot, if a real number x has a β-expansion that is distributed according to P_β[2], then $(x\beta^n)$ is u.d. modulo 1.

To see how this is useful to answer our question for Pisot bases, let us say we have a number x' such that $(x'\beta^n)$ is not u.d. modulo 1. Then, by Bertrand's theorem, its β-representation would have some block α whose average occurrences do not converge to $P_\beta(\alpha)$. We will show how to construct a polynomial-time martingale that succeeds by betting on that block, as is done in the integer base case. Yet, this cannot be done in a straightforward

[1] A martingale on alphabet Σ is a function $f : \Sigma^* \to \mathbb{Q}$ such that $\sum_{r\in\Sigma} f(\sigma r) = b \cdot f(\sigma)$, for all $\sigma \in \Sigma^*$. Its betting factors are the functions $d_r : \Sigma^* \to \mathbb{Q}$ for all $r \in \Sigma$ defined by $d_r(\sigma) = f(\sigma r)/f(\sigma)$. We say that f succeeds on an infinite sequence S if $\limsup_n f(S \restriction n) = \infty$.

[2] This is the analogue notion to being normal in base β.

manner, since the martingale condition as used in the algorithmic randomness literature assumes outcomes should be distributed according to the uniform measure (see footnote 1). If, however, expansions were supposed to obey some other distribution P, then a "fair" betting strategy f should have its martingale condition generalized to

$$f(a_1 \dots a_n) = \sum_{a \in \Sigma} P(a_1 \dots a_n a \mid a_1 \dots a_n) f(a_1 \dots a_n a). \tag{2}$$

Indeed, this definition captures the broader sense of *martingale* as it is used in probability theory. In this setting, not only may the probability of the next symbol be different from $|\Sigma|^{-1}$, it may also show all forms of conditional dependence on the preceding symbols. We will call a function f on Σ^* that satisfies (2) a *P-martingale*.

We then generalize Schnorr and Stimm's result to show that any sequence S is distributed according to a hidden Markov measure P if an only if no P-martingale with betting factors computed by a DFA succeeds on it. We use hidden Markov measures because they exhibit enough memorylessness to make them compatible with the memoryless structure of a DFA.

A second result by Bertrand, which says that P_β is hidden Markov when β is Pisot, allows us to apply our martingale construction to the β-expansion of x'.

Finally, we use the polynomial-time computability of the β-expansion and of P_β to show an integer base martingale that succeeds on x' can be constructed from our P_β-martingale, following the same ideas used in [3].

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A complexity lower bound based on software engineering concepts.

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We consider the problem of quantifier elimination in Effective Algebraic Geometry. This problem occupied the mind of mathematicians and programmers the last thirty years thinking efficient elimination algorithms. A key factor for the efficiency of elimination algorithms is the representation of polynomials. First elimination algorithms used polynomials given by coefficients (dense and sparse representation) and its computational complexity were doubly exponential, say $d^{n^{O(n)}}$ where d is the degree of the system and n is the number of variables to be eliminated. This complexity was later improved to $d^{O(n^2)}$ with the famous Effective Nullstellensatz. Later a new data structure produced considerable progress, polynomials were represented by arithmetic circuits evaluating them. This representation of polynomials reduced the complexity from $d^{O(n)}$ with hybrid implementations to the pseudo-polynomial $nd\delta^{O(1)}$ with the circuit-based elimination algorithm called Kronecker, in this last case δ is a semantic parameter which may be exponential in worst case.

The question is whether the actual asymptotic complexity of circuitbased elimination algorithms may be improved. The answer is no when elimination algorithms are constructed according to well known software engineering rules, namely applying Information Hiding and taking into account nonfunctional requirements. This conclusion is the result of a large investigation which culminated in a special computation model which captures algorithms produced by software engineering.

Our computation model evolved through time including first circuit based elimination algorithms with explicit circuit transformations (see [1] and [2] for details). In this model we obtained exponential complexity lower bounds. Now we ask what happen when the circuit representation of polynomials is replaced by another representation. This question leads to hide the representation obtaining an object-oriented elimination algorithm where the representation of polynomials remains encapsulated. Even for this kind of algorithms we can obtain an exponential complexity lower bound for the general task of quantifier elimination. This confirms, at least in our model, the exponential character of elimination.

Our computation model allows to give a new kind of complexity lower bounds and constitutes a surprising connection between Software Engineering and the theoretical fields of Algebraic Geometry and Computational Complexity Theory.

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On the spectra of recursive models of disintegrated strongly minimal theories

Uri Andrews and Steffen Lempp
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In 1978, Goncharov showed that there are strongly minimal theories where only the prime model has a recursive presentation. The spectrum of recursive models of a theory describes which models have recursive presentations. The general question is to characterize the spectra of recursive models of strongly minimal theories.

Since then, several other examples were given of strongly minimal theories where some models are recursive and others not. Most given examples involve disintegrated theories. A theory is disintegrated if the algebraic closure operator is given by the 2-variable formulae in the language. It is precisely this lack of structure, thus the combinatorial nature, of these theories that have allowed the constructions above.

We aim to classify the spectra of recursive models of disintegrated strongly minimal theories. A first step towards this goal was to classify the spectra of recursive models of disintegrated strongly minimal theories in finite languages.

Theorem.(A.-Medvedev)*If T is a disintegrated strongly minimal theory with a finite language, then there are exactly three possibilities for the spectrum of recursive models of T. Either every model is recursively presentable, no model is recursively presentable, or only the prime model is recursively presentable*

In the infinite language case, there is more recursion theoretic content to manage. We have determine the spectrum of recursive models of the strongly minimal theories in (infinite) binary languages.

Theorem.(A.-Lempp)*There are exactly 7 spectra of recursive models of disintegrated strongly minimal theories in binary languages.*

We make the following conjecture, with some evidence:

Conjecture.(A.-Lempp)*For every n, there are only finitely many spectra of recursive models of disintegrated strongly minimal theories in n-ary languages.*

I will describe our work on the above theorems as well as our hope to prove the conjecture.

Set theory

Set theory

Invited Talks.

Katetov order on MAD families

Carlos Martínez Ranero
Universidad de Concepción, Chile

Katetov ordering on almost disjoint families was introduced by Garcia-Ferreira and Hrusak in an attempt to classify them. We answered one of the basic questions by consistently constructing a MAD family maximal in this order. This is an ongoing project with many fundamental problems open.

Category forcings and generic absoluteness: steps towards a "complete" axiom system for set theory

Matteo Viale
Mathematical Department of Torino University, Italy.

Forcing is the most efficient method to produce independence results in set theory.

Woodin's work has shown that for problems formulated in second order number theory, forcing becomes a powerful tool to prove theorems and actually in the theory ZFC+ large cardinals gives a complete semantic for second order number theory with respect to first order derivability. We generalize Woodin's completeness result to a very large fragment of third order number theory with respect to an extension of ZFC+ large cardinals enriched with strong forcing axioms. Forcing axioms are usually formulated as strengthenings of the Baire category theorem and assert that for large classes of compact Haussdorff spaces X the intersection of $\aleph_1$-many open dense sets of X is non empty. Recently we've been able to give a formulation of these forcing axioms in the language of categories and to isolate according to this new characterization a forcing axiom (MM^{+++}) for which the mentioned generalization of Woodin's result to third order number theory holds with respect to the theory ZFC+large cardinals $+MM^{+++}$.

Relevant preprints are available on my webpage: http://www.personalweb.unito.it/matteo.viale/

Embedding Boolean algebras and Banach spaces

Christina Brech
Departamento de Matemática, Universidade de Sao Paulo, Brazil .

In this talk we will consider the problems of the existence of an embedding between two given Boolean algebras or two given Banach spaces and will try to understand how different these problems are. In the Banach spaces' context, an embedding can be an isomorphic embedding or an isometric embedding. Whenever an embedding from a Boolean algebra A into a Boolean algebra B exists, there exists also an isometric embedding from the Banach space C(S(A)) into the Banach space C(S(B)), where C(S(A)) stands for the Banach space of the continuous real-valued functions defined on the Stone space of A, with the supremum norm. Hence, there exists as well an isomorphic embedding. We prove that the other way round is in general not true and we shall discuss for which classes it is true. We also consider the problems of the existence of universal objects in these three classes and discuss whether these problems might be equivalent.

Contributed Talks.

Cardinal transfer theorem in admissible logics I

Kinrha Aguirre De la Luz and Cecilia Hernández Dominguez
Universidad Autónoma Metropolitana Unidad Iztapalapa, México.

Let M be a countable admissible set. We will prove the following gap-1 cardinal transfer theorem in M-logic, an admissible fragment of the infinitary logic $\mathcal{L}_{\omega_1\omega}$: given uncountable cardinals κ, λ, with λ singular, a first order countable language $\mathcal{L}$ with at least one unary predicate symbol U, and an $\mathcal{L}$-structure $\mathfrak{A} = \langle A, U^{\mathfrak{A}}, ...\rangle$, where $|A| = \kappa$, $|U^{\mathfrak{A}}| = \kappa$, we shall find an $\mathcal{L}$-structure $\mathfrak{B} = \langle B, U^{\mathfrak{B}}, ...\rangle$ such that $\mathfrak{B} \equiv_M \mathfrak{A}$, $|B| = \lambda$ and $|U^{\mathfrak{B}}| = \lambda$. The proof involves set theory and infinitary logics issues.

This work splits in two parts: combinatorial constructions (using squares), and model theoretical results (appeling to GCH). The first one will be presented in this talk.

Cardinal transfer theorem in admissible logics II

Cecilia Hernández Dominguez and Kinrha Aguirre De la Luz
Universidad Autónoma Metropolitana Unidad Iztapalapa, México.

Let M be a countable admissible set. We will prove the following gap-1 cardinal transfer theorem in M-logic, an admissible fragment of the infinitary logic $\mathcal{L}_{\omega_1\omega}$: given uncountable cardinals κ, λ, with λ singular, a first order countable language $\mathcal{L}$ with at least one unary predicate symbol U, and an $\mathcal{L}$-structure $\mathfrak{A} = \langle A, U^{\mathfrak{A}}, ...\rangle$, where $|A| = \kappa$, $|U^{\mathfrak{A}}| = \kappa$, we shall find an $\mathcal{L}$-structure $\mathfrak{B} = \langle B, U^{\mathfrak{B}}, ...\rangle$ such that $\mathfrak{B} \equiv_M \mathfrak{A}$, $|B| = \lambda$ and $|U^{\mathfrak{B}}| = \lambda$. The proof involves set theory and infinitary logics issues.

This work splits in two parts: combinatorial constructions (using squares), and model theoretical results (appeling to GCH). The second one will be presented in this talk.

On n-stationary sets

Joan Bagaria
ICREA and Universitat de Barcelona, Spain.

A subset S of a regular uncountable cardinal κ is 0-stationary if it is unbounded. And it is $n1$-stationary if for every $m \leq n$ and every m-stationary subset T of κ, there exists some $\alpha \in S$ where it m-reflects, i.e., $T \cap \alpha$ is m-stationary in α. Thus, S is 1-stationary if and only if it is stationary, in the usual sense. But the existence of 2-stationary sets has already some large-cardinal consistency strength. We present some recent results on n-stationary subsets of regular cardinals. In particular, we shall look at (1) the connections between n-stationarity and second-order indescribability in the constructible universe L, (2) the ideals associated to non n-stationary sets, and (3) the consistency strength of n-stationarity.

Generalizations of Gowers' FIN_k Theorem to multiple operations

Dana Bartosova
Universidade de Sao Paulo, Brazil.

We generalize Gowers' FIN_k Theorem to include not only one tetris operations, but k-many operations that behave as the identity up to some value, and as tetris above. With further extensions of the theorem, we are able to obtain a dual Ramsey theorem for the class of finite directed rooted trees. Combining this result with projective Fraïssé theory of Irwin and Solecki, we compute the universal minimal flow of the group of homeomorphisms of a one-dimensional continuum know as the Lelek fan. This is a joint work with Aleksandra Kwiatkowska.

Large cardinals limits of other large cardinals

Franqui Cárdenas
Universidad Nacional de Colombia, Bogotá, Colombia.

We present in this talk different version of the following result by Menas [1]: If κ is measurable cardinal and a limit of strongly compact cardinals. Then κ is a strongly compact. We observe the cases κ a Woodin cardinal and weakly compact cardinal limit of strongly unfoldable cardinals and the case κ strongly unfoldable cardinal limit of Woodin cardinals or strong cardinals.

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Dynamic sets and Modality over the time

Marisa Farusi
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In these pages, we are interested in modeling in the style of ZF Set Theory dynamic sets, i.e., entities that have the same identity but change over time, using the Modal Logic in a temporal sense and Justification Logic.

Universal submeasures and ideals

David Meza-Alcántara and Michael Hrusak
Universidad Michoacana de San Nicolás de Hidalgo, México.

We concerned the following general question: Let $\mathcal{M}$ be a class of ideals on ω. Is there an ideal I in $\mathcal{M}$ such that J is isomorphic to some restriction of I, for all $J \in \mathcal{M}$? Such an ideal will be called *universal* for $\mathcal{M}$. We deal this question for two classes: F_σ-ideals and analytic P-ideals. The key was to use the well known characterizations of these classes by Mazur and Solecki respectively, in terms of lower semicontinuous submeasures.

Ramsey properties, parity functions and selectors for equivalence relations

Carlos A. Di Prisco
Instituto Venezolano de Investigaciones Científicas, and Universidad de Los Andes, Bogotá.

A parity function is a function $f : 2^\omega \to 2$ such that whenever $x, y \in 2^\omega$ satisfy that $|\{n \in \omega : x(n) \neq y(n)\}| = 1$ then $f(x) \neq f(y)$. The existence of a parity function is equivalent to the failure of a partition property that is a weakening of the Ramsey property for subsets of the collection $\omega^{[\infty]}$ of infinite subsets of ω, and it implies the existence of certain sets of real numbers without the Baire property. We will examine the interrelations between some partition properties for sets of real numbers, the existence of transversals for certain equivalence relations and the existence of parity functions. We describe how to construct a model of ZF where there is no parity function but there are non-Baire sets, and a model of ZF where there is no transversal for the equivalence relation E_0 and there is a parity function. This work has been done in collaboration with A. R. D. Mathias and S. Todorcevic and it is closely related to work by S. Geschke, R. Lubarsky and M. Rahn.

Model theory

Model theory

Invited Talks.

Model theory of the Cuntz algebra O_2.

Isaac Goldbring
University of Illinois at Chicago, USA.

The Cuntz algebra O_2 is the universal C^* algebra generated by two (orthogonal) isometries whose sum is the identity. By remarkable results of Kirchberg and Kirchberg-Phillips, O_2 has come to play a serious role in the classification programme for (simple, separable, nuclear) C^* algebras. In this talk, I will show how O_2 also plays an important role in the model theory of C^* algebras. In particular, I will relate a conjecture of Kirchberg to the statement that O_2 is existentially closed (in the appropriate continuous signature for C^* algebras) and to model-theoretic forcing. I will end with some open questions about nonstandard models of the theory of O_2. This talk represents joint work with Thomas Sinclair.

Diophantine undecidability and uniform boundedness of rational points.

Xavier Vidaux
Universidad de Concepción, Chile.

We are interested in the general problem of classifying the extensions of Presburger arithmetic with respect to positive existential undecidability (Presburger showed in 1929 that $(\mathbb{Z}; 0, 1, +, =)$ has decidable full theory). We consider the particular, nevertheless significant case, when Presburger language is extended by a unary relation which is the range of a univariate polynomial. More precisely, given an arbitrary integer-valued polynomial $F(t)$, we consider the structure $\mathbb{Z}_F = (\mathbb{Z}; 0, 1, +, =, R)$, where R is the range of $F(t)$. If F has degree ≤ 1, then the full theory of $\mathbb{Z}_F$ is trivially decidable by Presburger's theorem (the positive existential theory is easily seen to be decidable without any use of Presburger's theorem). The first-order theory of $(\mathbb{N}; 0, 1, +, =, R)$ has been proven to be undecidable by H. Putnam and J. R. Büchi in the sixties for any F of degree at least 2, by showing that multiplication is definable. In the positive existential situation though, the problem is open in all cases when F has degree ≥ 2, unless some conjecture is assumed. Building on an original idea by J. R. Büchi in order to define multiplication in a positive existential way, P. Vojta showed in 1999 that $\mathbb{Z}_{t^2}$ has undecidable positive existential theory under Bombieri-Lang conjecture for surfaces. In 2012, H. Pasten obtained a similar definability result for $\mathbb{Z}_{t^k}$ for any $k \geq 2$ under the ABC conjecture. Using a new method, we show positive existential undecidability of $\mathbb{Z}_F$ for any F of degree at least 2 under the Caporaso-Harris-Mazur conjecture: the bound in Faltings' theorem (Mordell conjecture) should only depend on the genus of the curve and on the degree of the number field considered.

This work was partially supported by the Chilean Fondecyt research projects 1090233 and 1130134. This is joint work with Hector Pasten (partially supported by an Ontario Graduate Scholarship).

Issues in introducing logic and model theory to freshmen

Vinicius Cifu Lopes
Universidade Federal do ABC, Brazil

Doing anything relevant in model theory requires a good deal of mathematical knowledge. On the other hand, first-year college students who seek undergraduate research in logic must be welcomed. This requires the advisor to coach them to overcome their lack of mathematical background, which depends overall on the current scenario in high-school education. While it is possible to plan for a long-term learning schedule, starting with truth tables and some basic boolean algebra, one might ask for a straight quick introduction into predicate calculus, theories versus structures, and definability theory. The same concern applies to graduate students who have previously focused on engineering math and have had no contact with formal logic or abstract algebra. In this talk, we will discuss whether a solution is desirable, necessary, and possible, and will review an attempt in Brazilian education to bring formal mathematics and axiomatic set theory to early school curriculum.

Ordered theories where every infinite definable subset has interior.

Alfredo Dolich
Kingsborough Community College, CUNY, USA

Let R be an expansion of a divisible ordered Abelian group and let $T = Th(R)$. If we assume that T satisfies a strong form of not having the independence property we are quickly led to the condition that in any model of T any infinite definable subset of the line has non-empty interior. In this study we take this condition as axiomatic and consider its consequences on the properties of definable sets of any arity in models of T. We will review the initial motivation for this study and then discuss various desirable properties possessed by definable sets in models of such theories.

Stability, WAP, and Roelcke-precompact Polish groups.

Itaï Ben Yaacov
Université Claude Bernard - Lyon 1, France.

In joint work with T. Tsankov we study a (yet other) point at which model theory and dynamics intersect. On the one hand, a (metric) $\aleph_0$-categorical structure is determined, up to bi-interpretability, by its automorphism group, while on the other hand, such automorphism groups are exactly the Roelcke precompact ones. One can further identify formulae on the one hand with Roelcke-continuous functions on the other hand, and similarly stable formulae with WAP functions, providing an easy tool for proving that a group is Roelcke precompact and for calculating its Roelcke/WAP compactification. Model-theoretic techniques, transposed in this manner into the topological realm, allow one to prove further that if R(G) = W(G) then G is totally minimal.

Contributed Talks.

O-asymptotic classes of finite structures

Dario Garcia
Universidad de los Andes - Colombia

In [MS-2008], Macpherson and Steinhorn developed the notion of 1-dimensional asymptotic classes, which are classes of finite structures with a notion of measure and a dimension theories coming from the study of the size of definable sets. Specifically, they define

Definition 1. *Let $\mathcal{L}$ be a first order language, and $\mathcal{C}$ be a collection of finite $\mathcal{L}$-structures. Then $\mathcal{C}$ is a* 1-dimensional asymptotic class *if the following hold for every $n \in \mathbb{N}$ and every formula $\varphi(x, \overline{y})$, where $\overline{y} = (y_1, \cdots, y_m)$:*

1. *There is a positive constant C and a finite set $E \subseteq \mathbb{R}^{>0}$ such that for every $M \in \mathcal{C}$ and $\overline{a} \in M^m$, either $\varphi(M, \overline{a}) \leq C$ if $\varphi(M, \overline{a})$ is non-empty, or for some $\mu \in E$,*

$$||\varphi(M, \overline{a})| - \mu|M|| \leq C|M|^{1/2}$$

2. *For every $\mu \in E$, there is an $\mathcal{L}$-formula $\varphi_\mu(\overline{y})$ such that for all $M \in \mathcal{C}$, $\varphi_\mu(M^m)$ is precisely the set of $\overline{a} \in M^m$ with*

$$||\varphi(M, \overline{a})| - \mu|M|| \leq C|M|^{1/2}$$

The seminal example of these classes is the class of finite fields, for which these conditions appear as a remarkable theorem of Chatzidakis, Macintyre and Van den Dries. With this definition, which is a condition on the definable sets in only one variable, they can obtain results about the control of the size of definable sets in many variables as well as results concerning the behavior of the infinite ultraproducts of structures in such classes. For instance, they show that if every ultraproduct of the class $\mathcal{C}$ is strongly minimal, then $\mathcal{C}$ is a 1-dimensional asymptotic class. Furthermore, they prove that every ultraproduct of a 1-dimensional class is supersimple of U-rank 1.

An easy example of a class of finite structures which is not a 1-dimensional class is the class of all finite totally ordered sets, which fails property (1) because the formula $x < y$ can pick out an arbitrary proper initial segment of a structures as a varies. However, the only definable sets in the structures of this class (and in their ultraproducts) are finite unions of intervals and points implying that the structures involved are *$\mathcal{O}$-minimal.*

$\mathcal{O}$-minimality and its variants are properties that give a good structure theories for infinite ordered structures. Our aim here is to isolate conditions on classes of finite structures to get nice asymptotic properties, melding ideas of asymptotic classes and $\mathcal{O}$-minimality.

With this idea in mind, we propose a definition of *$\mathcal{O}$-asymptotic classes* as an adaptation of the definition of 1-dimensional asymptotic classes in the context of totally ordered structures.

In this talk I will present the definition O-asymptotic classes and talk about the main examples. I will also present the proof that shows that the ultraproducts of O-asymptotic classes are NTP_2 and superrosy of U-thorn-rank 1. Finally, I will mention a (coarse) cell decomposition theorem that can be obtained, as well as other possible ideas for further research.

Small Index Property and Reconstruction of Classes from Automorphisms Groups.

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The problem of "reconstructing a structure M from its automorphism group Aut(M)" has led to deep interaction between logic, topology, descriptive set theory and group theory, historically. Works of Lascar, Hodges, Shelah among others have reframed the problem as a problem of reconstructing the first order theory of a saturated structure. Results from descriptive set theory have been useful in the countable case, other ideas are useful in the uncountable cases. The "Small Index Property" has isolated a specific instance of "topology reduced to group theory" for the action of the group.

In joint work with Zaniar Ghadernezhad, we study versions of the Small Index Property for homogeneous models in abstract elementary classes, and we generalize the issue of reconstruction/interpretation to those contexts.

Temporal individuals in temporal structures

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The concept of an *individual* is commonly situated at the most ontologically basic level of a theory.

In this way, the general form of a model-theoretic structure $\mathcal{M}$ is standardly defined as a tuple

$$\langle A, R^{\mathcal{M}}, \dots, f^{\mathcal{M}}, \dots, c^{\mathcal{M}}, \dots \rangle,$$

with the universe A and the interpreted non-logical symbols of the underlying language. A is a non-empty set of individuals, which are the basic (first-order) entities of the structure.

In this talk, I shall outline an alternative perspective on individuals, according to which the elements of A are not individuals, but *individual occurrences*. Individuals are then understood at a higher level as particular sets of individual occurrences governed by the *individuality relation* $\equiv$.

Let us be given a (for our purposes first-order) language $\mathcal{L}$ with signature Ω. The logical symbols of $\mathcal{L}$ are as usual, together with the binary *sameness* relation $=$, which stands for the standard equality relation for identity (see below). But in our setting, the individuality relation $\equiv$ will feature in the list of binary relations of Ω.

The signature of $\mathcal{L}$ includes constant, as well as function and relation symbols with the relevant arity-encoding. In addition to $\equiv$, we find among the function symbols of Ω, a set $F_i = \{\omega_1^i, \omega_2^i, \dots\}$ of unary function symbols ω_j^i, for each natural number $n \in N$.

$\mathcal{L}$ is interpreted by means of *temporal structures* $\mathcal{M}_t$ (of signature Ω) whose universe is the non-empty set $\mathcal{O}$ of *individual occurrences*. We have in particular $N \subset \mathcal{O}$ (though one could also work with a two-sorted structure, with universes N and $A = \mathcal{O} \setminus N$ —i.e., of 'ordinary'-individual occurrences).

The interpretation of an ω_j^n in $\mathcal{M}_t$ is an *injective* function $\omega_{\mathbf{j}}^{\mathbf{n}} : I_n \to \mathcal{O}$, where $I_n = \{p \in N : p \leqslant n\}$.

The interpretation of $=$ in (any arbitrary) $\mathcal{M}_t$ is the set of pairs of elements of $\mathcal{O}$ such that they stand for the same individual occurrence. That is, $=$ is the analogous of the standard logical relation of equality. In particular, we have in the present account $\forall x(x = x)$ as a logical truth (for x a variable ranging over $\mathcal{O}$), which we shall call *self-sameness*.

The interpretation of $\equiv$ in $\mathcal{M}_t$ is an *equivalence* relation $\equiv$ over $\mathcal{O}$, and it is such that for all ω-functions ω, ω' and p in both $dom(\omega)$ and $dom(\omega')$, if we have $(\omega(\mathbf{p}), \omega'(\mathbf{p})) \in \equiv$, then $(\omega(\mathbf{p}), \omega'(\mathbf{p})) \in\ =$.

We call an $\mathcal{M}$-*individual*, or just individual for short, a family $\iota_m = \{\omega_{\mathbf{k_1}}^{\mathbf{1}}, \omega_{\mathbf{k_2}}^{\mathbf{2}}, \dots, \omega_{\mathbf{k_m}}^{\mathbf{m}}\}$ such that for every $\mathbf{i}, \mathbf{j} \leqslant \mathbf{m}$, and for all $\mathbf{p} \in dom(\omega_{\mathbf{k_i}}^{\mathbf{i}}), \mathbf{q} \in dom(\omega_{\mathbf{k_j}}^{\mathbf{j}})$, we have $(\omega_{\mathbf{k_i}}^{\mathbf{i}}(\mathbf{p}), \omega_{\mathbf{k_j}}^{\mathbf{j}}(\mathbf{q})) \in \equiv$. Finally, we say that $\omega_{\mathbf{k_i}}^{\mathbf{i}} \in \iota_m$ is an *individual of rank* i, for each $\mathbf{i} \leqslant \mathbf{m}$. It follows that any individual ι_m is an individual of rank $m+1$, and conversely.

Let us list some basic facts about sameness and identity in $\mathcal{M}_t$. In the following, we assume $dom(\omega) = I_n$ for an arbitrary n, and $\mathbf{p}, \mathbf{q} \in I_n$ (indices are omitted for clarity of notation).

1. $\mathcal{M} \vDash \forall p\, [\omega(p) = \omega(p)]$
2. $\mathcal{M} \vDash \forall p\, \forall q\, [p = q \Leftrightarrow p \equiv q]$
3. $\mathcal{M} \vDash \forall p\, \forall q\, [\omega(p) = \omega(q) \Leftrightarrow p = q]$

4. $\mathcal{M} \not\models \forall p \forall q\, [\omega(p) \equiv \omega(q) \Leftrightarrow p \equiv q]$
5. $\mathcal{M} \not\models \forall p \forall q\, [\omega(p) = \omega(q) \Leftrightarrow \omega(p) \equiv \omega(q)]$

(1) is the statement of self-sameness in $\mathcal{M}_t$.

(2) says that for the specific case of N, the notions of sameness and identity coincide, i.e., each individual occurrence is an individual, and vice versa. The intuitive reason is that natural numbers (and mathematical objects in general) are not subject to different occurrences: they are the same objects in each of their 'appearances'. This indicates that the standard conception of identity *qua* sameness is in fact suitable for mathematics. However, this is not the case for objects of the 'real world' in general, which undergo continuous change through time.

(3) follows immediately from the injectivity of ω, and says that two individual occurrences are the same occurrence if and only if they are related to the same natural number under ω. But (4) shows that the analogous result for identity fails to hold. Indeed, we may have $\omega(p) \equiv \omega(q)$ and $p \neq q$ —and thus $p \not\equiv q$ by (2). (5) is a generalization of (4), and states that sameness is not equivalent to identity: we may come upon different occurrences of the same individual.

The motivation of such an approach is hinted at through the very notion of an *occurrence*, which enables us to deal with a temporal notion of individual. This comes in contrast e.g. with standard modal tense structures, in which the basic (periodic or pointwise) units of time make up distinct structures (i.e., points of evaluation) by themselves —we have time-structures instead of temporal structures—; but also with standard temporal structures in which time units are themselves the elements of investigation —we have time-individuals instead of temporal individuals. In the present account, we deal with temporal individuals inside temporal structures.

Indeed, we may construe N as a set of time units, and a pair $(\mathbf{p}, \mathbf{a}) \in \omega$ as an individual occurrence $\mathbf{a}$ at time $\mathbf{t_p}$. An individual is then described as a set of individual occurrences governed by the individuality relation, i.e. such that they are occurrences of the same individual. Equivalently, an individual of rank p is the set of individual occurrences of that individual up to time $\mathbf{t_p}$.

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Definability and universal algebra

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In our talk we will introduce several (semantical) characterizations of definability, each concerning a specific format of first order formulas, e.g., definability by conjunctions of atomic formulas. We will show aswell how these characterizations can be applied to address interesting questions in the realm of universal algebra. For instance, to the definability of principal congruences and term interpolation results akin to the theorems of Baker and Pixley [1, 2].

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The model theory of modules of a C^* -algebra

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We study the theory of a Hilbert space H as a module for a unital C^*-algebra A from the point of view of continuous logic. We give an explicit axiomatization for this theory and describe the structure of all the representations which are elementary equivalent to it. Also, we show that this theory has quantifier elimination and we characterize the model companion of the incomplete theory of all non-degenerate representations of A, whose models are called *generic*. Finally, we show that for generic representations, there is an homeomorphism between the space of types of norm less than 1 in this model companion, and the space of quasistates of the C^*-algebra A .

Non-classical logics

Non-classical logics

Invited Talks.

Rational simplicial geometry and projective MV-algebras

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The variety of MV-algebras is the algebraic semantic of Łukasiewicz infinite valued logic. We refer the reader to [6] for the elementary theory of MV-algebras and to [10] for more advanced topics. An *MV-algebra* is an algebraic structure $(M, \oplus, \neg, 0)$, where $(M, \oplus, 0)$ a commutative monoid, $\neg$ is a unary operation satisfying $\neg\neg x = x$, $x \oplus \neg 0 = \neg 0$, and $\neg(\neg x \oplus y) \oplus y = \neg(\neg y \oplus x) \oplus x$. The real unit interval $[0, 1] \subseteq \mathbb{R}$ can be made into an MV-algebra $[0, 1]_{MV} = ([0, 1], \oplus, \neg, 0)$ where $x \oplus y = \min\{x + y, 1\}$ and $\neg x = 1 - x$. Chang's completeness theorem [6, Theorem 2.5.3], states that $[0, 1]_{MV}$ generates the variety of MV-algebras.

An MV-algebra M is *(regular) projective* if whenever $\psi\colon A \to B$ is a surjective homomorphism and $\phi\colon M \to B$ is homomorphism, there is a homomorphism $\theta\colon M \to A$ such that $\phi = \psi \circ \theta$. In this paper we provide a geometric description of finitely generated projective MV-algebras, thus solving the sixth problem in the list of open problems presented by Mundici in [10, Section 20.3].

In [9], Mundici proved that unital ℓ-groups (abelian groups equipped with a translation invariant lattice-order and with a distinguished strong unit) are categorically equivalent to the variety of MV-algebras. Since (regular) projective objects are preserved under categorical equivalences, the main result in this paper provides also a complete description of projective unital ℓ-groups. Baker [1] and Beynon [2] proved that an ℓ-group G is finitely generated projective if and only it is finitely presented. We will see that the characterisation of projective unital ℓ-groups is far more delicate.

To present our result we need the following definitions. A simplex S is said to be *rational* if the coordinates of its vertices are rational numbers. A set $P \subseteq \mathbb{R}^n$ is said to be a *rational polyhedron* if there are rational simplexes $T_1, \ldots, T_l \subseteq \mathbb{R}^n$ such that $P = T_1 \cup \cdots \cup T_l$. Given a rational polyhedron $P \subseteq [0, 1]^n$. we let $\mathcal{M}(P)$ denote the set of all continuous functions $f\colon P \to [0, 1]$ having the following property: there are finitely many linear (in the affine sense) polynomials $p_1, \ldots, p_m$ with integer coefficients, such that for all $x \in [0, 1]^n$ there is $i \in \{1, \ldots, m\}$ with $f(x) = p_i(x)$. The set $\mathcal{M}(P)$ is the universe of a subalgebra of the MV-algebra $([0, 1]_{MV})^P$. Therefore, $\mathcal{M}(P)$ carries a natural structure of MV-algebra, where the operations are defined pointwise from $[0, 1]_{MV}$.

In [4] and [5] the author and D. Mundici studied of projective MV-algebras (unital ℓ-groups). We proved that finitely generated projective MV-algebras are isomorphic to $\mathcal{M}(P)$ where the rational polyhedron $P \subseteq [0, 1]^n$ is a special kind of retract of $[0, 1]^n$ (for some $n = 1, 2, \ldots$) called $\mathbb{Z}$-retract. Since $\mathbb{Z}$-retract are continuous retractions of cubes, they are trivially *contractible*, that is, homotopically equivalent to a point (see for example [7, Chapter 0])).

Another important property of $\mathbb{Z}$-retracts is that they are *strongly regular* (see [5, Definition 3.1]). In [3, Theorem 4.17], we observed how strong regularity is connected with the notion of anchored polytopes defined by Jeřábek in [8]. In this paper we will first present a simple description of strongly regular polyhedron. More precisely, we prove that a rational polyhedron P is strongly regular if and only if for each $v \in P \cap \mathbb{Q}^n$ there exist $w \in \mathbb{Z}^n$ and $\varepsilon > 0$ such that the convex segment $\mathrm{conv}(v, v + \varepsilon(w - v))$ is contained in P. Using this result an equivalent presentation of [5, Theorem 4.5] is as follows.

Theorem 1. *If the MV-algebra A is finitely generated and projective, then there exist $n \in \{1, 2, \ldots\}$ and a rational polyhedron $P \subseteq [0, 1]^n$ such that $A \cong \mathcal{M}(P)$ and P satisfies the following conditions:*

(i) *P is contractible,*

(ii) *$P \cap \{0, 1\}^n \neq \emptyset$, and*

(iii) *for each $v \in P \cap \mathbb{Q}^n$ there exist $w \in \mathbb{Z}^n$ and $\varepsilon > 0$ such that the convex segment $\mathrm{conv}(v, v + \varepsilon(w - v))$ is contained in P.*

This paper is devoted to proving the converse of Theorem 1. In the special case when the rational polyhedron P is a finite union of 1-simplexes, the converse was already proved in [5, Corollary 5.4], but the general case has remained open until now.

The proof of the main result of this paper requires a combination of tools from algebraic topology, polyhedral and simplicial geometry.

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Classifying strongly finite logics in the Leibniz hierarchy

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One of the main topics of Abstract Algebraic Logic [4, 5] is the study of the ***Leibniz operator***, i.e., a particular map $\boldsymbol{\Omega}^{\boldsymbol{A}} \colon \mathcal{P}(A) \to \mathrm{Co}\boldsymbol{A}$, which can be defined for every algebra $\boldsymbol{A}$ and associates a specific congruence with any subset of the universe of $\boldsymbol{A}$. One of the major applications of this study has been to exploit the order-theoretic and set-theoretic behaviour of the operator over the deductive filters of a logic $\mathcal{L}$, in order to capture interesting facts about its own definability and that of the truth predicate, in models of $\mathcal{L}$. This was essentially discovered by Blok and Pigozzi [1] for algebraizable logics, and their work and that of other scholars (Czelakowski, Herrmann, Jansana, Raftery) on this topic has given rise to a whole hierarchy, called the ***Leibniz hierarchy*** [3, 6], in which logics are classified by means of properties of the Leibniz operator which determine how nicely the Leibniz congruences and the truth predicates can be described in models of the logic.

Along this talk we shall focus on the classification of ***strongly finite logics***, i.e., logics determined by a finite set of finite logical matrices, within the Leibniz hierarchy. The starting point is the observation that, under the assumption that $\mathcal{L}$ is a strongly finite logic, it is possible to determine algorithmically the precise location of $\mathcal{L}$ within the Leibniz hierarchy. More precisely we build:

- An algorithm that determines whether $\mathcal{L}$ is ***protoalgebraic*** or not and in the positive case it provides a set of protoimplication formulas $\Delta(x, y)$ and a set of congruence formulas with parameters $\Delta(x, y, \overline{z})$ for $\mathcal{L}$.
- An algorithm that determines whether $\mathcal{L}$ is ***equivalential*** or not and in the positive case it provides a set of congruence formulas $\Delta(x, y)$ for $\mathcal{L}$.
- An algorithm that determines whether $\mathcal{L}$ is ***truth-equational*** or not and in the positive case it provides a set of defining equations $\boldsymbol{\tau}(x)$ for $\mathcal{L}$.
- An algorithm that determines whether $\mathcal{L}$ is ***weakly algebraizable*** or not and in the positive case it provides a set of congruence formulas with parameters $\Delta(x, y, \overline{z})$ and a set of defining equations $\boldsymbol{\tau}(x)$ for $\mathcal{L}$.

- An algorithm that determines whether $\mathcal{L}$ is ***algebraizable*** or not and in the positive case it provides a pair of structural transformers $\boldsymbol{\rho}(x, y)$ and $\boldsymbol{\tau}(x)$ which witness the algebraizability of $\mathcal{L}$.

The idea which lies behind these algorithms is that of building the free algebra $\boldsymbol{Fm}_{\mathsf{V}}(k)$ with a suitable finite number of generators k over the variety V, generated by the algebraic reducts of the matrices defining the logic $\mathcal{L}$. Since $\boldsymbol{Fm}_{\mathsf{V}}(k)$ is always finite, we can check step by step if there is a subset of its universe, which yields a set of congruence formulas $\Delta(x, y, \overline{z})$ for $\mathcal{L}$. An analogous strategy can be used in the quest for a set of defining equations $\boldsymbol{\tau}(x)$ for $\mathcal{L}$, since equations can be though as pairs of elements of $\boldsymbol{Fm}_{\mathsf{V}}(k)$.

In the second part of the talk we focus on special classes of logics for which stronger, finer results on the classification, especially in the algebraizable case, can be obtained. More precisely, the idea is that of considering logics determined by algebras which enjoy growing degrees of polynomial completeness, which will turn out to correspond to degrees of expressibility of the logically meaningful facts, captured by the Leibniz hierarchy. In particular, we prove that for logics determined by a single matrix $\langle \boldsymbol{A}, F\rangle$ such that $F \notin \{\emptyset, A\}$, whose algebraic reduct $\boldsymbol{A}$ is a non-trivial ***quasi-primal algebra***, the following conditions hold:

$$\texttt{protoalgebraic} = \text{having theorems} \tag{3}$$

$$\texttt{truth-equational} = \text{having theorems} F \text{ equationally definable} \tag{4}$$

$$\texttt{algebraizable} = \text{truth-equational} F \cap B \neq B \text{ f.a. non-trivial } \boldsymbol{B} \in \mathbb{S}(\boldsymbol{A}). \tag{5}$$

In particular (3) and (4) imply that, within this framework, truth-equational logics coincide with the weakly-algebraizable ones. Moreover, it is possible to prove that every algebraizable logic, whose equivalent algebraic semantics coincides with $\mathbb{V}(\boldsymbol{A})$, is of the kind (5) (recall that $\boldsymbol{A}$ is assumed to be quasi-primal).

Further results can be obtained if the algebra $\boldsymbol{A}$ under consideration is ***primal*** (and non-trivial). More precisely, under these assumptions, it is possible to prove that every logic determined by a matrix $\langle \boldsymbol{A}, F\rangle$ such that $F \notin \{\emptyset, A\}$ is algebraizable with equivalent algebraic semantics $\mathbb{V}(\boldsymbol{A})$ and, conversely, that every algebraizable logic, whose equivalent algebraic semantics is $\mathbb{V}(\boldsymbol{A})$, is determined by a matrix $\langle \boldsymbol{A}, F\rangle$ for some $F \notin \{\emptyset, A\}$.

Finally, we say that a finite algebra $\boldsymbol{A}$ is ***ubiquitous algebraizable*** if the matrix $\langle \boldsymbol{A}, F\rangle$ determines an algebraizable logic of $\mathbb{V}(\boldsymbol{A})$ for every $F \notin \{\emptyset, A\}$. We address the problem of whether the property of being ubiquitous algebraizable is actually equivalent to primality or not, which, for the moment, is still an open problem. Nevertheless, this property is surprisingly close to that of being primal: if $\boldsymbol{A}$ is ubiquitous algebraizable, then $\boldsymbol{A}$ is simple and is the only subdirectly irreducible member of $\mathbb{V}(\boldsymbol{A})$, has not proper subalgebra, no automorphism apart from the identity map and generates a point-regular variety. In particular, this means that $\mathbb{V}(\boldsymbol{A})$ is congruence modular and n-permutable for some $n \in \omega$. Even if the problem in general is still open, we are able to provide a positive solution in the case of two-element algebras.

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F-structures and matrix semantics for Logics of Formal Inconsistency

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In 1977, M. Fidel and D. Vakarelov independently proposed a semantics for D. Nelson's logic in terms of a novel class of algebraic structures now called *twist structures*. Also in 1977, Fidel obtained for the first time the decidability of da Costa's paraconsistent systems C_n by using certain algebraic-relational semantical structures now called *Fidel structures* (or **F**-structures). Being non-truth-functional, logics in the hierarchy C_n as well as their generalization within the hierarchy of paraconsistent logics known as *Logics of Formal Inconsistency* (**LFI**s), do not have non-trivial logical congruences, and so they bravely resist to the semantical analysis based on standard tools, like categorial or algebraic semantics. This is why the development of alternative semantical techniques for this kind of **LFI**s is deemed necessary.

In this talk we introduce a new class of semantics in terms of **F**-structures, as well as a matrix semantics in terms of a new class of twist-like structures called *swap structures*. Both semantics characterize **mbC**, the weaker logic in the hierarchy of **LFI**s. The equivalence between both semantics is proved directly, and a new proof of the decidability of **mbC** in terms of swap structures defined over the 2-element Boolean algebra is also obtained. The same results will be obtained for several extensions of **mbC**, including da Costa's system C_1.

This is a joint work with Walter Carnielli (CLE-UNICAMP).

Some adjunction involving the spectrum functor

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Let $\mathcal{H}_f$ be the category of finite Heyting algebras. The assigment to each H in $\mathcal{H}_f$ of its (finite) poset of prime filters defines a functor Spec : $\mathcal{H}_f^{op} \to Pos_f$. It is well known that this functor is part of a categorical equivalence (a particular case of Esakia duality). If we restrict Spec to the full subcategory of $\mathcal{H}_f$ whose objects are the prelinear algebras (finite Gödel algebras), we get the equivalence Spec : $\mathcal{G}_f^{op} \to \mathcal{F}_f$, where $\mathcal{F}_f$ is the category of finite forests with p-morphisms as arrows.

On the other hand, it is also well known that a (finite) poset is isomorphic to the prime spectrum of a unital abelian ℓ-group if and only if it is a (finite) forest. It is the case that Spec : $(\ell - \mathcal{A}^u)_f s^{op} \to \mathcal{F}_f$ is a functor from the category of unital abelian ℓ-group with finite spectrum to the category of finite forests. It is hence natural to ask whether this functor is part of a categorical equivalence or not; and if not, if it has right or left adjoints.

In this talk we shall answer this question and state some related ones.

Contributed Talks.

Another proof of Mundici's equivalence

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Following a suggestion of E. Dubuc, we derive Mundici's result using the basic fact that every MV-algebra is a subdirect product of chains, and Chang's equivalence between the category of MV-chains and totally ordered abelian groups with a distinguished strong unit. D. Mundici established an equivalence between the category of MV-algebras and that of lattice ordered abelian groups with a distinguished strong unit, extending Chang's equivalence. To do this, given an MV-algebra A, he introduced an elaborate construction of "good sequences" of elements of A. On the other hand, Chang's result for a MV-chain A uses a very simple and intuitive construction defining a structure of l-group on the cartesian product $\mathbb{Z} \times A$.

Given an MV-algebra A, we define $L(A)$ as the free abelian group generated by A, modulo an equivalence relation. Each element of $L(A)$ is interpreted as a function on A's prime spectrum, with values on the subdirect product of the chain groups coming from the MV-chains A/P. Two functions are equivalent if they coincide on every prime. Formal sums are interpreted at each P using the sum of Chang's representation. We show that the functor L is equivalent to Mundici's Γ functor.

Finally, in the special case of semisimple MV-algebras we show some practical examples.

An alternative definition of quantifier on four valued Lukasiewicz algebras

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In [4] Halmos presents the monadic Boolean algebras as the algebraic counterpart of the monadic predicate calculus. Several generalizations of monadic algebras have been obtained for different classes of algebras associated to non-classic logics. Particularly, the monadic four-valued Łukasiewicz algebras have been studied, for example in [1, 2, 3], as a four-valued Łukasiewicz algebra endowed with an existential quantifier, here called standard existential quantifier. In this work, an alternative notion of existential quantifier on four-valued Łukasiewicz algebras is considered. This notion of quantifier arise as a generalization of ternary existential quantifiers on three-valued Łukasiewicz algebras introduced by A. Petrovich in [7]. The class of four-valued Łukasiewicz algebras endowed with this existential quantifier, denoted by $\exists_{2/3}$, is defined and studied. It is shown that $\exists_{2/3}$ is interdefinable with the standard existential quantifier on a four-valued Łukasiewicz algebra. Some connections between $\exists_{2/3}$ and existential quantifiers defined on bounded distributive lattices and Boolean algebras are given. Finally, a completeness theorem for the monadic four-valued Lukasiewicz predicate calculus corresponding to standard universal quantifier is given. Then, a second completeness theorem for the monadic four-valued Lukasiewicz predicate calculus corresponding to the dual of the quantifier $\exists_{2/3}$ is proven, using the fact that both quantifiers are interdefinable.

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RI-rigs and their representation

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A *rig*, as introduced in [S], is a commutative "ring without negatives", that is, having two commutative monoid structures $(0,)$ and $(1, \cdot)$ related by the distributive laws $0 = a \cdot 0$ and $a \cdot ba \cdot c = a \cdot (bc)$. If the equation $1x = 1$ is satisfied it will be said that the rig is *integral.* In any integral rig the additive monoid defines a semilattice and so, an underlying partial order. Each function $a \cdot (\_)$ is monotone with respect to this order and the integral rig will be called *residuated* if all these monotone maps have right adjoints. For brevity, integral residuated rigs will be called *RI-rigs.*
A RI-rig is *connected* if the following axiom

$$(ab = 1 \text{ and } a \cdot b = 0) \Rightarrow (a = 1 \text{ or } b = 1)$$

holds.
We prove that every RI-rig A in **Sets** can be represented as the algebra of points of a connected RI-rig (in a topos built from A).We also discuss how to derive a more standard result (in the spirit of those presented in [P]) in terms of global sections of continuous functions, using the fact that the toposes used in our proof are equivalent to toposes of local homeos.

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A tableau for set-satisfiability for extended fuzzy logic BL

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Classical satisfiability of a propositional formula in a structure (model) is understood as the truth value of the formula relative to an assignments of truth values to propositional atoms. We say that a formula is satisfiable if such an assignment exists, in which it is true. In mathematical fuzzy logic a formula has a truth value, which is a value from an interval [0,1], where 0 is the absolute false, 1 is the absolute true, and other values indicate a partial truth. We can also consider a wider notion of satisfiability than classical. Following [2] and [9], we can work with $\mathcal{K}$-satisfiability, where $\mathcal{K} \subseteq [0, 1]$. A formula ψ is $\mathcal{K}$-satisfiable if its truth value belongs to $\mathcal{K}$ under some assignment of values from [0,1] to atoms. To be able to define set-satisfiability rigorously, we need an exposition of a fuzzy logic.

Basic fuzzy logic BL is a propositional fuzzy logic, in which formulas of BL are written with propositional atoms, $\bar{0}$ (falsum) and $\bar{1}$ (verum), joined by $\&;, \rightarrow, \vee, \wedge, \leftrightarrow$. (see [5]) Logic $\mathrm{BL}_{\triangle\sim}$ is BL with additional unary connectives Delta connective $\triangle$ and the involutive negation $\sim$. The semantics of the logic is defined as follows. A t-norm $\star$ (also called a residuated t-norm if it has a residuum) is a function defined on $[0, 1]^2$ with values in $[0, 1]$ such as it is associative, commutative, non-decreasing with neutral element 1, and its residuum (if it exists) is a function $\Rightarrow: [0, 1]^2 \rightarrow [0, 1]$ satisfying $x \star z \leq y$ iff $z \leq x \Rightarrow y$. We define the function $\triangle : [0, 1] \rightarrow [0, 1]$ by $\triangle x = 1$ if $x = 1$, otherwise $\triangle x = 0$ for all $x \in [0, 1]$. Given $\mathrm{BL}_{\triangle\sim}$ formulas ψ, φ, the assignment V of propositional atoms to elements of [0,1], a residuated t-norm $\star$, we inductively define $\star$-evaluation $V_\star$ as $V_\star(\bar{0}) = 0$, $V_\star(\bar{1}) = 1$, $V_\star(\psi\&;\varphi) = V_\star(\psi) \star V_\star(\varphi)$, $V_\star(\psi \rightarrow \varphi) = V_\star(\psi) \Rightarrow V_\star(\varphi)$, $V_\star(\psi \vee \varphi) = \max\{V_\star(\psi), V_\star(\varphi)\}$, $V_\star(\psi \wedge \varphi) = \min\{V_\star(\psi), V_\star(\varphi)\}$, $V_\star(\triangle\psi) = \triangle V_\star(\psi)$, $V_\star(\sim \psi) = 1 - V_\star(\psi)$. Our work on satisfiability will be based on a continuous residuated t-norm.

We can now formally define $\mathcal{K}$-satisfiability. A formula ψ is $\mathcal{K}$-satisfiable if there exists a continuous residuated t-norm $\star$ and an assignment V of atoms to [0,1], for which $V_\star(\psi) \in \mathcal{K}$. In this paper we will build a model for a finite set of formulas of $\mathrm{BL}_{\triangle\sim}$ that is $\mathcal{K}$-satisfiable in this model. To achieve this, we will use tableau methods as a semantic proof system. The idea is based on decomposition theorem (see [3], [5], [6], [8]), by which a continuous t-norm (and thus its residuum) is expressed as a family of Product and Łukasiewicz components. The set of formulas are translated into tableau formulas and the complement of the set $\mathcal{K}$ with respect to [0,1] is expressed as a union of subintervals. The latter enables to create finite branches of the tableau, i.e. finite sequences of tableau formulas, in which the next element of the sequence has fewer symbols of interpreted connectives. The branches are extended using some rules that we defined.

The main result is that the tableau for the set of formulas for the subset $\mathcal{K}$ of truth values [0,1] is open iff this set of formulas is $\mathcal{K}$-satisfiable. The tableau calculus presented in this paper enables for $\mathcal{K} \in [0,1]$ and a finite set of formulas Ψ of $\mathrm{BL}_{\triangle\sim}$ to find a continuous t-norm $\star$ and an assignment V of propositional atoms to [0,1] such that $V_\star(\psi) \in \mathcal{K}$ for all $\psi \in \Psi$, or alternatively to show that such a model does not exist.

The axiomatization of $\mathrm{BL}_{\triangle\sim}$ has been shown to be finitely strong standard complete with respect to a continuous residuated t-norm with the additional unary connectives $\triangle, \sim$ (see [4]). If $\mathcal{K} = [0,1)$ and our tableau closes, that is there is no such a model for which the finite set of formulas are not tautologies, the set of formulas are provable in $\mathrm{BL}_{\triangle\sim}$.

It is worth noticing that there exist tableau calculi or other proof systems for BL or Łukasiewicz logics that demonstrate that a formula is a tautology or that it is not (see [1], [7], [10], [11], [12], [13]). The advantage of our tableau over the ones exiting in the literature is three-fold, (1) in case that the formula is not a tautology, it constructs a countermodel, (2) we can show that a formula is $\mathcal{K}$-satisfiable for $\mathcal{K} \subseteq [0,1]$, (3) our calculus tackles a set of formulas from an extended logic BL with additional unary connectives.

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Soundness of a temporal semantics for BL

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Many-valued propositional logics generalise classical logic through the addition of new truth values between absolute falsity and absolute truth.

Hájek's *Basic Fuzzy Logic* BL provides a complete axiomatization of the theorems common to all $[0,1]$-valued continuous t-norm based propositional systems, in which the conjunction is interpreted by a continuous t-norm and the implication by its associated residuum. BL admits a wide spectrum of schematic extensions, including the well-known Łukasiewicz extension. Aguzzoli, Bianchi and Marra proved in [1] that BL-formulas can be thought as modal formulas over an appropriate flow of time, where the logic of each instant is Łukasiewicz. The main result of their work states that such temporal semantics is *sound* and *complete* with respect to BL. Briefly,

- A *temporal flow* is a pair $\langle T, L\rangle$ where T is a poset and $L : T \to \mathbb{N}$ is a function.
- A *temporal assignment* over BL-formulas is a function $v : FORM \times T \to [0,1]$ such that $v(\bot, t) = 0$, $v(\varphi \& \psi, t) = v(\varphi, t) \odot v(\psi, t)$ and

$$v(\varphi \to \psi, t) = \begin{cases} 1 & \text{if } v(\varphi, t') \leq v(\psi, t') \ \forall t' \geq t \\ v(\varphi, t) \to v(\psi, t) & \text{if } v(\psi, t) < v(\varphi, t) < 1 \\ & \text{and } v(\psi, t') = 1 \ \forall t' > t \\ v(\psi, t) & \text{otherwise} \end{cases},$$

  where $t' \in T$.
- $\langle T, L\rangle \models \varphi$ if $v(\varphi, t) = 1$ for all temporal assignment v and all $t \in T$.
- A class K of temporal flows is *sound* (for BL) if, for any formula φ, $\vdash_{BL} \varphi$ implies $\langle T, L\rangle \models \varphi$ for all $\langle T, L\rangle \in K$.

BL-algebras are the algebraic counterpart of BL. They are commutative, integral and bounded residuated lattices that satisfy prelinearity and divisibility. Because of prelinearity, the fundamental structures in the study of BL-algebras are totally ordered BL-algebras (BL-chains). BL-chains are closely related to totally ordered Wajsberg hoops: *every totally ordered BL-algebra is an ordinal sum of a family totally ordered Wajsberg hoops.*

Jipsen and Montagna extended the notion of ordinal sum to integral GBL-algebras, which is class of algebras that contains BL. They introduced a generalised construction that they called *poset sum* (see [3]). This construction was revised in [2] under the name of *poset product* in order to study BL-algebras representation. Since our work will deal with BL-algebras, the latter definition will be regarded here. Let P be a poset and let $\{A_i\}_{i \in P}$ be a collection of commutative, integral and bounded residuated lattices. We can assume that all A_i share the same neutral element 1 and the same minimum element 0. The *poset product* is the algebra denoted and defined as follows:

$$A = \bigotimes_{i \in P} A_i = \{a \in \prod_{i \in P} A_i : \forall j \in P \ (\forall k \in P \ (a_j < 1 \wedge j < k \implies a_k = 0))\}.$$

Monoid and lattice operations $\wedge_A$, $\vee_A$ and $*_A$ are defined pointwise; and the residuum is

$$(a \to_A b)_i = \begin{cases} a_i \to_i b_i & \text{if } a_j \leq b_j \text{ for all } j < i \\ 0 & \text{otherwise} \end{cases}.$$

It is worthwhile to note that every BL-algebra can be embedded into the poset product of MV-chains (see [2]).

The proof of soundness of temporal flows (more precisely, the class of *prelinear* flows) for BL given in [1] is carried out in stages. A proof in terms of totally ordered temporal flows is suggested there. Our purpose is to provide an alternative proof, in a single step, which will be based on the poset product representation. Both proofs share a basic scheme: to build a BL-algebra B and a valuation $w : FORM \to B$ such that $w(\varphi) < 1_B$ for a BL-formula φ that is not modeled by a temporal flow and then, to invoke the algebraic completeness theorem.

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On some categories of involutive residuated lattices with ideals

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In [4] we have studied the relationship between the infinite-valued logic Ł of Łukasiewicz and a new system that we call Ł$^\bullet$. The models of the logic Ł$^\bullet$ are the objects of the algebraic category MV$^\bullet$, which is dually equivalent to the category MV of MV–algebras by means of the functor K$^\bullet$ whose adjoint is a functor $\mathcal{K}$ (see [1, 2]). Also, we define a more general deductive system whose equivalent algebraic semantics is a variety of involutive residuated lattices in which there is a unary map κ that plays an important role.

In particular, in [4] we found a link between the Lindenbaum algebra of Ł, A_{MV}, and that of $Ł^\bullet$, $A_{\mathrm{MV}^\bullet}$, by means of an isomorphism $\kappa(A_{\mathrm{MV}^\bullet}) \cong A_{\mathrm{MV}}/I$, for some ideal I of A_{MV}. This fact and related work by L. Monteiro and I. Viglizzo [3, 5] motivate us to define an extension of the functor $\mathrm{K}^\bullet$. The objects of the domain category of the new functor are pairs (A, I), where A is an integral commutative residuated lattice which has an involution and I is an ideal of A.

In this talk we shall study some properties of the extended functor $\mathrm{K}^\bullet$, in particular we show that it defines a categorical equivalence.

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Improving memory performance for mimp-graph based theorem provers

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This work presents the initial results on the construction of a [semi] automated theorem prover for Minimal Implicational Propositional Logic ($\mathbf{M}_\rightarrow$) based on a graph representation of proofs denoted by mimp-graphs, as pointed out by [1, 4, 5].

The choice for $\mathbf{M}_\rightarrow$ was motivated by its validity of computational complexity be PSPACE-Complete. In fact, this fragment can polynomially simulate Classical, Intuitionistic and full Minimal Logic and any propositional logic with a Natural Deduction system with the subformula property [1].

Mimp-graphs are directed acyclic graphs that allow reuse by the usage of references to point to different occurrences of a formulas inside a proof. In this work we explore how this reuse of formulas leads to a reduced size of the proofs regarding to traditional ways to present them in Natural Deduction and Sequent Calculus. We show that the use of mimp-graphs to represent proofs presents a compression factor inversially proportional to the logarithm of the size of the conclusion of the proof.

We also present a theorem prover, based on sequent calculus, for $\mathbf{M}_\rightarrow$ that applies the aforementioned fact to improve its efficiency and memory usage during the proof search. Eventually, we theoretically compare the impact of such a approach when applied for a specific class of formulas of $\mathbf{M}_\rightarrow$, known from [2, 3] to have exponentially lower bound for normal proofs.

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Fibring by Functions over Matrix Logics

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The technique of combining logics known as *fibring* was introduced by Dov Gabbay in [3], with the aim of combining logics having Kripke semantics, such as intuitionistic and modal logic. The underlying idea of this technique is, roughly speaking, the following: given two logics $\mathcal{L}_1$ and $\mathcal{L}_2$, with Kripke semantics Kr_1 and Kr_2 respectively, a new logic $\mathcal{L}_1 \circledast \mathcal{L}_2$ is defined over the language obtained from the union of the connectives of both languages. In order to evaluate, namely, a modality of $\mathcal{L}_i$ applied to formulas of $\mathcal{L}_j$ $(i \neq j)$, pairs of functions are considered. For every pair, the first function associates, to each world of a model of Kr_1, a world of a model of Kr_2,while the second one acts from Kr_2 to Kr_1. By considering all the possible pairs of such functions, the hybrid formulas can be evaluated, and $\mathcal{L}_1 \circledast \mathcal{L}_2$ is a weak extension of $\mathcal{L}_1$ and $\mathcal{L}_2$, which means that the tautologies of the original logics are also tautologies of the fibred one.

An interesting adaptation of fibring using the framework of Category Theory was introduced in [4] (see also [1]).

Continuing a research started in [2], we present in this paper an adaptation to matrix logics of Gabbay's technique of fibring. Our method, called "fibring by functions", focuses on the combination of two arbitrary logics defined by a matrix semantics, obtaining a new matrix logic defined in the combined language which includes connectives of both original logics.

From now on, the algebra of propositional formulas generated by a set $MATHCALV$ of propositional variables from a signature C will be denoted by $L(C)$, and $X \uplus Y$ will denote the disjoint union of the sets X and Y.

Definition. Let $\mathcal{L}_i = \langle C_i, \models_{M_i} \rangle$ be a logic defined over signature C_i, presented by means of a logical matrix $M_i = (A_i, D_i)$, for $i = 1, 2$ (as usual, the same symbol will be used for connectives and operators interpreting them). A pair of functions $(\lambda_1, \lambda_2) \in {A_2}^{A_1} \times {A_1}^{A_2}$ is called a *fibring pair.* Given a fibring pair (λ_1, λ_2), consider the functions $*_i : A_1 \uplus A_2 \longrightarrow A_j$ for $i \neq j$ defined as follows: $*_i(a) = a$, if $a \in A_j$, and $*_i(a) = \lambda_i(a)$, if $a \in A_i$. The *fibred matrix* $M_{(\lambda_1,\lambda_2)} = (A_1 \uplus A_2, D_1 \uplus D_2)$ over $C_1 \uplus C_2$ is defined as follows: for $i \neq j$, if $c \in C_i$ is k-ary and $\vec{a} \in (A_1 \uplus A_2)^k$ then $c(\vec{a}) := c(*_j(a_1), \ldots, *_j(a_k))$. The logic associated to $M_{(\lambda_1,\lambda_2)}$ will be denoted by $[\mathcal{L}_1 \circledast \mathcal{L}_2]_{(\lambda_1,\lambda_2)}$.

The relationship between $[\mathcal{L}_1 \circledast \mathcal{L}_2]_{(\lambda_1,\lambda_2)}$ and $\mathcal{L}_i$, for $i = 1, 2$, can be stated.

Proposition. Let $\mathcal{L}_i = \langle C_i, \models_{M_i} \rangle$ be a nontrivial matrix logic (for $i = 1, 2$). Then, for each fibring pair (λ_1, λ_2), $[\mathcal{L}_1 \circledast \mathcal{L}_2]_{(\lambda_1,\lambda_2)}$ is a weak conservative extension of both $\mathcal{L}_1$ and $\mathcal{L}_2$. That is, $\models_{M_i} \alpha$ iff $\models_{M_{(\lambda_1,\lambda_2)}} \alpha$, for every $\alpha \in L(C_i)$ and $i = 1, 2$.

However, $[\mathcal{L}_1 \circledast \mathcal{L}_2]_{(\lambda_1,\lambda_2)}$ is not necessarily a strong extension of the given logics. The following result can be proved:

Proposition. Let $\mathcal{L}_1$, $\mathcal{L}_2$ as above, and let (λ_1, λ_2) be an admissible fibring pair. That is: $\lambda_i(a) \in D_j$ iff $a \in D_i$, for every $a \in A_i$ and $i \neq j$. Then $[\mathcal{L}_1 \circledast \mathcal{L}_2]_{(\lambda_1,\lambda_2)}$ is a strong conservative extension of both $\mathcal{L}_1$ and $\mathcal{L}_2$. That is, $\Gamma \models_{M_i} \alpha$ iff $\Gamma \models_{M_{(\lambda_1,\lambda_2)}} \alpha$, for every $\Gamma \cup \{\alpha\} \subseteq L(C_i)$ and $i = 1, 2$.

Categorial fibring can be also defined by *sharing* connectives, that is, two connectives of different signatures are identified in the combined signature, and so in the fibred logic (see [4] and [1]). The notion of "identification of connectives" is a bit more subtle in the present framework:

Definition. Given two connectives c_1, c_2 of the same arity in an arbitrary signature C, we say that the formulas β and γ of $L(C)$ are (c_1, c_2)-*associated* if γ is obtained from β by some replacements of the connective c_1 by c_2, and vice versa.

Definition. Let $\mathcal{L}_1$, $\mathcal{L}_2$ as above. We say that a fibring pair (λ_1, λ_2) *identifies the connectives* c_1 *and* c_2 (were $c_i \in C_i$ is k-ary) if the fibring $[\mathcal{L}_1 \circledast \mathcal{L}_2]_{(\lambda_1,\lambda_2)}$ verifies the following: for every pair of (c_1, c_2)-associated formulas β, γ in $L(C_1 \uplus C_2)$ and for every k-tuple $\vec{a} = (a_1, \ldots, a_k) \in (A_1 \uplus A_2)^k$, it holds:

$$\beta(\vec{a}) \in D_1 \uplus D_2 \ \text{ iff } \ \gamma(\vec{a}) \in D_1 \uplus D_2.$$

Theorem. Let $\mathcal{L}_1$, $\mathcal{L}_2$, c_1 and c_2 as in the previous definition, and let (λ_1, λ_2) be a fibring pair satisfying the following conditions:

(*a*) (λ_1, λ_2) is an admissible pair;

(*b*) $\lambda_1|_{A_1/c_1} : A_1/c_1 \longrightarrow A_2/c_2$ is an isomorphism, where A_i/c_i denotes the reduct of A_i to the signature $\{c_i\}$, for $= 1, 2$;

(*c*) $\lambda_2|_{A_2/c_2} = (\lambda_1|_{A_1/c_1})^{-1}$.

Then, (λ_1, λ_2) identifies c_1 with c_2.

The question of determining if the previous conditions are also necessary is still open.

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Connectives and schemas

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We aim to present some results concerning intuitionistic, dual intuitionistic and some weaker logics. In particular, due to its being a central question, we consider whether adding certain connectives is or is not a conservative extension of the given logic. In case it is not, we study how much force the connective has, i.e. which is the non-conservative schema derived when the connective is added. We examine both the case of propositional logic and first order logic as there are cases where the addition of a connective is conservative over propositional logic but not over first order logic. We also consider the question whether the addition of a connective may be achieved without adding a new rule, i.e. just adding axioms.

We mostly consider examples, such as the Smetanich constant, the S connective of Kuznetsov, the strongest anticipator of Humberstone, the duals of intuitionistic conditional and negation, a variation of the Δ connective in fuzzy logic and some other modal connectives. We also strive to get some general results.

A rapprochement between algebra and logic

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Algebraic proof mechanisms based on handling polynomials over finite fields (the *polynomial ring calculus*, PRC) have been investigated since 2001, with some interesting results. From a historical perspective, the idea of using formal polynomials as algebraic proof procedures refurbish the intuition about the intimacy between algebra and logic, already implicit in some ideas by Leibniz, Boole, De Morgan, Peirce, Schröder, Hilbert and Tarski, among others. The Russian mathematician I. I. Zhegalkin already proposed in 1927 a method to translate and decide propositions from Whitehead and Russell's *Principia Mathematica* by means of translating sentences into polynomials with coefficients in the two-element field $\mathbf{Z}_2$ (see [Car07]for references). PRC consists of translating sentences into multivariable polynomials in the ring $\mathrm{GF}(p^m)[X]$ of polynomials with coefficients in the Galois field of order p^n, and propositional derivability is reduced to checking whether or not certain families of (finite) polynomial equations have solutions (reading truth-values as elements of the field).

PRC is applicable to several domains, as to monadic first-order logic, to finitely many-valued logics in general and to certain non-truth-functional logics (e.g., paraconsistent logics). Of especial interest are the cases of modal logics and full first-order logic (FOL). An extension of the method to the modal logics **K**, **KD**, **T**, **S4**, **S5** and to intuitionistic logic is investigated in [AAC11] and [AAC14].

Nonetheless, first-order logic requires infinite polynomials, based on specific algebraic domain of generalized series closed under products. Now, a fundamental problem relating logic and algebra is to specify in which sense a certain class of algebras corresponds to a given logical system. This seems to be more problematic for FOL than to other cases; as it is well known, the process of algebraization of FOL is intricate, and not thought to be really natural. As remarked in [Moo99], the logics considered from 1879 to 1923, such as the ones championed by Frege, Peirce, Schröder, Löwenheim, Skolem, Peano, and Russell, were generally richer than first-order logic. One of the reasons for such richness was the use of infinitely long expressions, specially by Peirce and Schröder.

Such infinitely long expressions, understandably, disappeared from contemporary first-order predicate calculus. The current, standard algebraic paradigms regarding FOL are the well-known polyatomic algebras of Paul Halmos and the cylindric algebras of Alfred Tarski, both started in the fifties.

However, despite their intrinsic interest, cylindric algebras and polyadic algebras can hardly be called 'natural': their conceptual difficulties and complex methods of proof put them far away from the intuitive character of Boolean algebras. I want to suggest that this disjuncture between first-order logic and its intended algebraic counterpart may be due to the reluctancy of modern logicians in using infinitely long expressions, as exemplified by expressing existential quantifiers (essentially infinitary objects) in cylindric algebras by means of the (finitary) operations Cx.

I argue that the representation of FOL by means of formal (infinite) polynomials in PRC, very much resembling Taylor series, not only offers a new proof method to first-order logic comparable to analytic tableaux, but can be legitimately regarded as an algebraic semantics (in the sense of [Riv09]), a counterpart to FOL with a higher degree of naturalness. PRC also offers an algebraic counterpart to certain paraconsistent logics that are not amenable to the Blok-Pigozzi method, as it is the case of various paraconsistent logics.

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Some general results on the translations between logics

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In the late twenties and early thirties of last century several results were obtained connecting different logics and theories. These results assumed the form of translations/interpretations of one logic/theory into another logic/theory. In 1925 Kolmogorov defined a translation from classical logic into minimal logic aiming to show that ?classical mathematics? can be translated into ?intuitionistic mathematics?. In 1929 Glivenko proved two fundamental results connecting provability in classical propositional logic to provability in intutionistic propositional logic: Glivenko 1: A is a classical propositional theorem if and only if $\neg\neg A$ is an intuitionistic propositional theorem. Glivenko 2: $\neg A$ is a classical propositional theorem if and only if $\neg A$ is an intuitionistic propositional theorem. In 1933, Gödel and Gentzen independently defined an interpretation of Classical/Peano Arithmetic (PA) into Heyting?s arithmetic (HA). A preparatory step in Gödel?s interpretation establishes that classical propositional logic cannot be distinguished from intuitionistic propositional logic with respect to theorems in the fragment $\{\neg, \wedge\}$. In 1933 Gödel also defined an interpretation of intuitionistic propositional logic into the classical modal logic S4. A minimum requirement for all these translations is that they preserve deducibility: Given two logics L1 and L2 and a translation T of L2 into L1, then $S \vdash_{L2} A$ if and only if $T[S] \vdash_{L1} T[A]$. The aim of the present paper is to show the following results concerning translation between logics and theories: [1] The first result establishes that given two logics L1 and L2 and a translation of L2 into L1, then, given any intermediate logic L3 between L1 and L2, the same translation can be used to translate L2 into L3. [2] In 1979, R. Statman showed a translation from Intuitionistic Propositional Logic into its implicational fragment. This reduction is polynomial and proves that Purely Implicational Minimal Logic is PSPACE-complete. The methods that Statman uses are based on proof-theory and Natural Deduction in Prawitz Style. The sub-formula principle for a Propositional Natural Deduction system NL for a logic L states that whenever α is provable from Γ, in L, there is a derivation of α from a set of assumptions $\{\delta_1, \ldots, \delta_k\} \subseteq \Gamma$ built up only with sub-formulas of α and/or $\{\delta_1, \ldots, \delta_k\}$. We show that any propositional logic L, with a Natural Deduction system that satisfies the sub-formula principle has a translation to purely minimal implicational logic. In fact, since this translations is polynomially bounded and L includes the purely minimal implicational logic, then L is PSPACE-complete. [3] The third result establishes that if T is a first order theory formulated in the language $\{\neg, \wedge, \rightarrow, \forall\}$ such that T is atomically stable (for every atomic sentence A of T it holds that T proves $A \leftrightarrow \neg\neg A$), then every theorem of T can be proved without the use of classical reasoning.

In fact, the translation defined in item [2] above can be used to obtain the result in [1]. In the final part of the paper I would like to discuss the topic of intermediate arithmetics. In a famous passage in his paper on the relation between intuitionistic and classical arithmetic, Gerhard Gentzen affirms that the difference between the two arithmetics is ?purely external?, and by this he means that the difference lies in the logic: the same arithmetical structure (axioms and/or rules) on top of classical logic produces classical arithmetic, while on top of intuitionistic logic produces Heyting?s arithmetic. This interesting idea could suggest that by choosing an intermediate logic L between classical logic and intuitionistic logic one would produce an intermediate L-arithmetic between classical arithmetic and intuitionistic arithmetic. But we know that this idea is not true: by choosing the intermediate logic of Constant Domains CD (intuitionistic logic $\forall x(A(x) \vee B) \rightarrow (\forall x A(x) \vee B)$, x not free in B) we do not produce a CD-arithmetic, but classical arithmetic itself instead. Obviously the same collapse is produced by any logic L between CD and classical logic. On the other hand, we also know that there are genuine intermediate arithmetics. Given that CD already establishes a ?transition point? in the area between classical and intuitionistic arithmetic, it would certainly be interesting to investigate if this transition point could be moved downwards and to what extend downward-collapses can be produced.

Representation by tuples of Lukasiewicz-Moisil algebras of order $n+1$ and applications

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In order to study non–classical logics, for many years, a wide range of algebraic structures have been considered and studied. The study of such algebras presents many similarities and analogies in both concepts and demonstrations. This may lead to think that there are not methods and principles more general than those. However, a different line of study have also been considered: it is possible to study the structure and meaning of algebraic models of certain non–classical logics by means of tuples. The method of tuples, also known as the method of twist structures, have shown to be simpler and illuminating. This technique can be displayed, for example, studying the Nelson logic in [1] and [3].

In this work, we apply the technique of tuples to the well–known Łukasiewicz–Moisil algebras of order $n+1$. With this representation, the theory of homomorphisms, filters and free algebras will be simplified significantly. Finally, a $n+1$-valued logic sound and complete with respect to this new semantic will be also presented.

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Monadic theta-valued Lukasiewicz-Moisil algebras without negation

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In 1954, P. Halmos introduced monadic Boolean algebras in [6] and in 1957, A. Monteiro and O. Varsavsky in [7] considered a generalization of monadic Boolean algebras and defined monadic Heyting algebras, which are deeply studied by Bezhanishvili. In 1997, A.V. Figallo and A. Ziliani introduced in [1] monadic distributive lattices as a natural generalization of monadic Heyting algebras. In [2] and [5], we determined two topological dualities for these algebras. In [3] we investigated a subvariety of monadic distributive lattices which we call strong monadic distributive lattices (sM–lattices). Our interest to study them derived from the fact that, to some extent, they are close to monadic Boolean algebras. Indeed, sM–lattices satisfy all the properties that hold in monadic Boolean algebras which do not involve the negation operation. More precisely, sM–lattices are monadic distributive lattices satisfying the identity: $\forall(x \vee \forall y) = \forall x \vee \forall y$.

In this article we introduce monadic and strong monadic $\theta-$valued Łukasiewicz–Moisil algebras without negation generalizing the notions of monadic distributive lattices and strong monadic distributive lattices, respectively. In addition, we develop a topological duality for each of these classes of algebras, extending the dualities that we obtained previously for monadic distributive lattices and strong monadic distributive lattices in [5] and [3], repectively, and for $\theta-$valued Łukasiewicz–Moisil algebras without negation in [4]. Among others results, from these dualities we determine propierties of these algebras, which allow us to state that the notions of monadic $\theta-$valued Łukasiewicz–Moisil algebras and strong monadic $\theta-$valued Łukasiewicz–Moisil algebras are equivalent.

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The Boolean algebra of central elements

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By a *variety with* 0 *and* 1 we understand a variety $\mathcal{V}$ for which there are 0-ary terms 0 and 1 such that

$$\mathcal{V} \vDash 0 = 1 \rightarrow x = y.$$

If $A \in \mathcal{V}$, then we say that $e \in A$ is a *central element* of A if there exists an isomorphism $\tau\colon A \to A_1 \times A_2$ such that $e \mapsto (0,1)$. We let $Z(A)$ and $FC(A)$ denote, respectively, the set of central elements and the set of factor congruences of A.

We say that $\mathcal{V}$ has the *determining property* (DP) if for every central element e there is a unique pair (θ, δ) of complementary factor congruences such that

$$e \equiv 0(\theta) \text{ and } e \equiv 1(\delta).$$

Thus DP is in some sense the most general condition guaranteeing that central elements have the whole information about the direct product decompositions in the variety. It is clear that DP is implied by the following definability condition.

($*$) There is a first-order a formula $\varphi(z,x,y)$ such that for every $A, B \in \mathcal{V}$ and $a, a' \in A$, $b, b' \in B$

$$A \times B \vDash \varphi((0,1),(a,b),(a',b')) \text{ iff } a = a'.$$

Also, we note that a simple application of Beth's definability theorem produces the implication DP $\Rightarrow$ ($*$). In [1] it is proved that DP and ($*$) are both equivalent to $\mathcal{V}$ having *Boolean factor congruences* (BFC).

If $\mathcal{V}$ has the DP, then every factor congruence θ has a unique factor complement θ^*, which implies that the maps

$$\begin{aligned} e amp;&\mapsto \theta^A_{0,e}, \\ \theta amp;&\mapsto \text{unique } e \text{ satisfying } e \equiv 0(\theta) \text{ and } e \equiv 1(\theta^*), \end{aligned}$$

are mutually inverse bijections between $Z(A)$ and $FC(A)$. Since

$$(FC(A), \vee, \cap, ^*, \Delta^A, \nabla^A)$$

is a Boolean algebra, we can define

$$\begin{aligned} e \vee^{Z(A)} f amp;&= \text{only } g \in Z(A) \text{ satisfying } \theta^A_{0,g} = \theta^A_{0,e} \vee \theta^A_{0,f}, \\ e \wedge^{Z(A)} f amp;&= \text{only } g \in Z(A) \text{ satisfying } \theta^A_{0,g} = \theta^A_{0,e} \cap \theta^A_{0,f}, \\ c^{Z(A)}(e) amp;&= \text{only } g \in Z(A) \text{ satisfying } \theta^A_{0,g} = \left(\theta^A_{0,e}\right)^*, \end{aligned}$$

to obtain a Boolean algebra $(Z(A), \vee^{Z(A)}, \wedge^{Z(A)}, c^{Z(A)}, 0, 1)$, which is naturally isomorphic to $(FC(A), \vee, \cap, ^*, \Delta^A, \nabla^A)$.

We study definability of basic operations on $Z(A)$, in particular the case when $\mathcal{V}$ has the *Fraser-Horn property.*

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Display-type calculi and their cut-elimination methatheorem

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The range of non-classical logics has been rapidly expanding, driven by influences from other fields which have opened up new opportunities for applications. The logical formalisms which have been developed as a result of this interaction have attracted the interest of a wider research community than the logicians, and their theory has been intensively investigated, especially with respect to their semantics and computational complexity.

However, most of these logics lack a comparable proof-theoretic development. More often than not, the hurdles preventing a standard proof-theoretic development for these logics are due precisely to some of their defining features which make them suitable for applications, such as e.g. their not being closed under uniform substitution, or the fact that (the semantic interpretations of) key connectives are not adjoints.

These difficulties caused the existing proposals in literature to be often ad hoc, not easily generalisable, and more in general lacking a smooth proof-theoretic behaviour. In particular, the difficulty in smoothly transferring results from one logic to another is a problem in itself, since these logics typically come in large families (consider for instance the family of dynamic logics), and hence proof-theoretic approaches which uniformly apply to each logic in a given family are in high demand (for an expanded discussion of the existing proof systems for dynamic epistemic logics, see [5, section 3]).

The problem of the transfer of results, tools and methodologies has been addressed in the proof-theoretic literature for the families of substructural and modal logics, and has given rise to the development of several generalisations of Gentzen sequent calculi (such as hyper-, higher level-, display- or labelled-sequent calculi).

In this talk we focus on the core technical aspects of a proof-theoretic methodology and set-up closely linked to display logic [2] and basic logic [1]. Instances of this set-up have appeared in [5] and [6] to account for (nonclassical versions of) Baltag-Moss-Solecki's dynamic epistemic logic. In ongoing work, this set-up is being applied to propositional dynamic logic [3], monotone modal logic [4], game logic, and linear logic.

The present set-up, which we refer to as *display-type calculi*, generalizes display calculi in two aspects: by allowing multi-type languages, and by dropping the full display property. The generalisation to a multi-type environment makes it possible to introduce specific tools enhancing expressivity, which have proved useful e.g. for a smooth proof-theoretic treatment of multi-modal and dynamic logics [6, 3]. The generalisation to a setting in which full display property is not required makes it possible to account for logics which admit connectives which are neither adjoints nor residuals [4].

One technical aspect which guarantees the cut elimination meta-theorem to go through for display-type calculi, even in the absence of the full display property, concerns the strengthening of the *separation* property (requiring principal formulas in introduction rules to appear in isolation) to the *visibility* property. Visibility requires *all active formulas* in introduction rules to occur in isolation. This property was recognized to be crucial for the cut elimination theorem of basic logic [1].

However, in the present set-up of display-type calculi, visibility is also weakened, in the sense that, in order to account for logics that are not closed under uniform substitution [5, 6], principal formulas in axioms are not required to occur in isolation.

In the proposed talk, we will illustrate the basic principles of the multi-type environment, and also how the above combination of weakenings, strengthenings of the separation property concurs to guaranteeing the cut elimination meta-theorem for display-type calculi.

Time permitting, we will also discuss some difficulties that still arise in the case of PDL and some ideas towards treating predicative logics.

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Emergence in defeasible reasoning: an adaptive logic perspective on the computational complexity of defeasible reasoning

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In this talk I will analyse the computational complexity of adaptive logics from the point of view of emergence.

Adaptive logics (ALs) are logics that formalize (ideally all kinds of) defeasible reasoning. They enrich a normal (i.e. monotonic, reflexive, transitive and compact) logic (the so called LLL of an AL) by assuming that formulas of a specific form (the abnormalities) are false whenever the premises do not prevent this. The set of consequences are semantically defined as the consequences that are true in those LLL-models that are minimal with respect to an ordering $<$ on the set of LLL-models of the premises. The ordering $<$ is defined by how many abnormalities the models verify: $M_1 > M_2$ iff M_1 verifies more abnormalities than M_2. The strategy of an AL determines how the word "more" in the last sentence is precisely interpreted.

The AL-consequence relations are in the worst case extremely complex. The set of consequences of a recursive premise set can be Σ^0_3-complete in the case of Reliability ALs or Finitary Minimal Abnormality ALs and even Π^1_1 in the case of full Minimal Abnormality ALs. Nevertheless ALs arguably accurately explicate many forms of defeasible reasoning actually used in the sciences and in everyday reasoning. One might conclude from this that the forms of defeasible reasoning captured by ALs are sometimes too complex for finitary agents. In other words: in certain situations there is no finitary way to get to know what the ideally rational conclusions are which one can defeasibly draw from a set of premises.

The dynamic proofs of ALs are, nevertheless, quite simple. These are proofs the lines of which are derived conditionally. In a typical AL-proof, the formulas that occur on lines are either considered derived in the proof (in case the line is not marked) or are considered not derived (in case the line is marked). A rather simple recursive marking rule determines the marking of lines. The proofs are dynamic: conclusions may come and go and come back again as the proof continues. The proofs aim to formalize defeasible reasoning processes. We thus obtain somewhat surprising conclusion that, although some ideal defeasible consequence relations seem too complicated for finitary agents, rational reasoning towards this ideal is perfectly feasible for finitary agents.

This phenomenon can be analysed as a form of emergence: There is a strong and complex form of validity that emerges form simple dynamic rules.

I clarify this with an analogy that stems from the works of C.S. Peirce (incidentally the first one to take defeasible reasoning seriously) (cf. CP 4.127). When he wants to explain the concept of a neighborhood of a (mathematical) point on a surface or on a line, he gives the example of a blot of black ink on a white surface. He asks the reader what the color is of the points on the actual borderline between the white surface and the black blot. He concludes that the color of the points in the immediate neighborhood of (i.e.–roughly speaking–the points indistinguishably close to) this borderline is undetermined. They both have the potentiality to be white and to be black (although every point is ultimately either white or black and not both).

In a similar way we can look at the precise borderline between actual AL-consequences and non-AL-consequences of a premise set in the mind of a rational finitary reasoner. The LLL-consequences are clearly AL-consequences (the IN-category), and the non-ULL-consequences, where ULL is the logic obtained by deleting all models that make abnormalities true, are clearly non-AL consequences (the OUT-category). For the other formulas, the dynamic AL proofs try to find out to what category they belong. At every stage of a proof in which a formula A occurs on a line, the marking in the proof gives us an estimation as to whether it is in the ideal AL-consequences (IN) or it is not (OUT). Given that the markings may come and go, the proof's estimation of the derivability of A oscillates between the IN-category and the OUT-category.

Nevertheless, given the way ALs are defined, the IN and OUT-categories are mutually disjoint and together exhaustive (there is after all one precise set of AL-consequences). The formulas that are in the final IN-category, are the ones for which there exists a (possibly infinite) stable proof. So the simple rules of the dynamic proofs determine exactly the very complex set of AL-consequences. That's why I claim that the set of ideal AL-consequences can be seen as a concept that emerges from the AL-proofs

Given the limitations of finitary agents, one cannot expect of them to reason in such a way that they come up with exactly those conclusions that are in the very complex ideal AL-consequences. However, such agents can be expected to reason dynamically with rules that warrant that their conclusions in the limit stabilise into the ideal consequences. During the reasoning process, the formulas for which one has not obtained clarity yet are epistemologically undetermined w.r.t. their derivability, but nevertheless they may belong to a provisional IN/OUT-category in the proof that has not yet stabilised.

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Validity in a paraconsistent framework

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The so called Validity Paradox can be used to argue that validity is an inconsistent notion that must be characterized in a paraconsistent framework. But, can paraconsistent positions express the concept of validity? Beall 2009 and Beall and Murzi 2012 claim that paraconsistent positions can't do that. We will show how it is possible to do so (both in dialetheist and non-dialetheist frameworks). We will adopt Toby Meadows? general strategy in order to give a (paracomplete, in Meadows system) predicate of validity. In Meadows unpublished, he adopts Kripke?s fixed point construction for a validity predicate. Its interpretation has a set of pairs of formulae as an extension, and another one as its anti-extension. There will be not formulae that belong to the intersection of both, but there will be pairs of formulae that are neither in the extension nor in the anti-extension. Meadows uses a jump operator that adopts a ?tolerant-strict? clause to determine the extension, and a ?strict-tolerant? ?or classical- clause to determine the anti-extension. The final interpretation of the validity predicate will be the minimal fixed point of the construction. But it will have some flaws. Meadows defines the consequence relation in terms of the validity predicate. And as the interpretation of the validity predicate won't be reflexive, nor monotonic nor transitive, neither will be its consequence relation. We can import Meadows? strategy to a paraconsistent framework. We will present versions of it in Ripley?s ?Strong-Tolerant? logic, and in LP. The validity predicate will be define in exactly the same way as Meadows does, and so the construction will be non-decreasing, and will eventually reach a fixed point. As the consequence relation of both systems is defined in an independent manner, the validity predicate won't represent exactly the consequence relation of neither of both logics. But both consequences relations will be reflexive and monotonic. Each of them has advantages and disadvantages. LP?s consequence relation is also transitive, but ST (expanded with a ?Meadows-like? validity predicate) can the consistently expanded with a truth predicate.

A Topological Duality for F-chains associated to Paraconsistent Logics

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In this work we present a topological representation for the subclass of F_ω-structures whose supports are chains. Briefly, F_ω-structures were defined by M.M. Fidel in [3], and are the basis of the algebraic-relational semantics for the paraconsistent Da Costa's logic C_ω (see [2]). A simplified characterization of F_ω-structures can be found in [4], which will be slightly adapted to the case of F_ω-chains, as follows:

Definition. A $\boldsymbol{F_\omega}$**-structure** is a pair $\langle L, \mathfrak{f}\rangle$, being $\langle L, \vee, \wedge, \rightarrow, 1\rangle$ a relatively pseudocomplemented lattice (R.P.L., from now on), and $\mathfrak{f} : L \to \wp(L)$ verifies, for every $x \in L$:
(F1) $\emptyset \subset \mathfrak{f}(x) \subseteq x^{\mathsf{T}}$
(F2) $\mathfrak{f}(y) \cap {\downarrow} x \neq \emptyset$, for every $y \in f(x)$.
(Here, ${\downarrow} x$ is the decreasing set of by x, and $x^{\mathsf{T}} := \{y \in L : y \vee x = 1\}$).
In addition, if L is a chain, the pair $\langle L, \mathsf{A}\rangle$ will be called a $\boldsymbol{F_\omega}$**-chain**.

Note that it can be demonstrated that every F_ω-chain can be characterized as a pair $\langle L, \mathsf{A}\rangle$, where L is a chain with greatest element 1, and $A \subseteq L$ satisfies:
(C) For every $x \in L$, $\mathsf{A} \cap {\downarrow} x \neq \emptyset$.
The set A will be often called a **down covering set of** L.
With this characterization in mind, we will show that the category of F_ω-chains has a dual equivalent category. This result can be developed in this way. First, we define a class of adequate ordered topological spaces. This definition has several common points with others in the literature (see [1] for example).

Definition. A $\boldsymbol{Ch_\omega}$**-space**, is 4-tuple $X = \langle X, \leq, \mathcal{B}, \mathcal{S}\rangle$ verifying:
(S1) $\langle X, \leq\rangle$ is a chain.
(S2) $\mathcal{B}$ is a base for a topology on X of compact and decreasing sets of X, and $\emptyset \in \mathcal{B}$.
(S3) For every x, $y \in X$ with $x \not\leq y$, exists $B \in \mathcal{B}$ such that $x \notin B$, $y \in B$.
(S4) If F is a closed subset of X and $\mathbf{T}$ is a non-empty family of $\mathcal{B}$ such that, for every $T \in \mathbf{T}$, $F \cap T \neq \emptyset$, then $F \cap \bigcap\{T : T \in \mathbf{T}\} \neq \emptyset$.

(S5) $\mathcal{S} \subseteq \mathcal{B}$, verifying that, for every $B \in \mathcal{B}$ there is $S \in \mathcal{S}$ such that $B \subseteq S$.

Theorem. *If* $X = \langle X, \leq, \mathcal{B}, \mathcal{S} \rangle$ *is a* Ch_ω*-space, then* $\mathbb{D}(X) = \langle D(X), \mathbb{A} \rangle$ *is a* F_ω*-chain, where* $D(X) := \{U : U^c \in \mathcal{B}\}$ *is the* ***dual of*** X *and* $\mathbb{A} = \{S : S^c \in \mathcal{S}\}$.

Theorem. *If* $L = \langle L, \mathsf{A} \rangle$ *is a* F_ω*-chain, then* $\mathbb{X}(L) = \langle X(L), \subseteq, \mathcal{B}_L, \mathcal{S}_L \rangle$ *is a* Ch_ω*-space, called* ***the dual space of*** L*, where* $\mathcal{B}_L = \{\varphi_L(x)^c : x \in L\}$ *and* $\mathcal{S}_L = \{\varphi_L(a)^c : a \in \mathsf{A}\}$.
So, from the previous results, every F_ω-chain can be represented by a Ch_ω-space, and vice versa.

Topological duality for F_ω-chains

Finally, we relate F_ω-chains and Ch_ω-spaces, at the level of categories.

Definition. The **category** $\mathfrak{C}$ consists of F_ω-chains as objects, being its morphisms (between F_ω-chains (L_i, A_i), $i = 1, 2$), the mappings $h : L_1 \to L_2$ that satisfy:

Definition. Let $L_1 = \langle L_1, \mathsf{A}_1 \rangle$ and $L_2 = \langle L_2, \mathsf{A}_2 \rangle$ be two F_ω-chains. A mapping $h : L_1 \to L_2$ is an **homomorphism** if it satisfies:
(H1) h is monotone.
(H2) $h(\mathsf{A}_1) \subseteq \mathsf{A}_2$.
(H3) for every $y \in L_2$, there exists $x \in L_1$ such that $h(x) \leq y$.

Definition. The **category** $\mathfrak{S}$ has, as $\mathfrak{S}$-objects, the Ch_ω-spaces, being the $\mathfrak{S}$-morphisms (between two Ch_ω-spaces $\langle X_1, \leq_1, \mathcal{B}_1, \mathcal{S}_1 \rangle$ and $\langle X_2, \leq_2, \mathcal{B}_2 \mathcal{S}_2 \rangle$), the functions $f : X_1 \longrightarrow X_2$ satisfying:
(A1) f is monotone.
(A2) $f^{-1}(T) \in \mathcal{B}_1$, for every $T \in \mathcal{B}_2$.
(A3) $f^{-1}(T) \in \mathcal{S}_1$, for every $T \in \mathcal{S}_2$.

Definition. Let $\langle X_1, \leq_1, \mathcal{B}_1, \mathcal{S}_1 \rangle$ and $\langle X_2, \leq_2, \mathcal{B}_2, \mathcal{S}_2 \rangle$ be two Ch_ω-spaces. We say that a map f from X_1 into X_2 is a Ch_ω**-function** if it satisfies:
(A1) f is monotone.
(A2) $f^{-1}(T) \in \mathcal{B}_1$, for every $T \in \mathcal{B}_2$.
(A3) $f^{-1}(T) \in \mathcal{S}_1$, for every $T \in \mathcal{S}_2$.

With the previous definitions in mind, the main result to be showed is the following:

Theorem. *The categories* $\mathfrak{C}$ *and* $\mathfrak{S}$ *are dually equivalent*, where $\mathfrak{C}$ is the category with F_ω-chains as objects and homomorphisms as arrows and $\mathfrak{S}$ is the category with Ch_ω-spaces as objects and Ch_ω-functions as arrows.

The previous results will be exemplified by means of bounded and non-bounded F_ω-chains.

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Two-valued Logics for Naive Truth Theory

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It is part of the current wisdom that the Liar and similar semantic paradoxes can be taken care of by the use of certain non-classical multivalued logics. The rough idea, due to Kripke, is to use three-valued models with

$\{1, \frac{1}{2}, 0\}$ as the set of semantic values. In [3] he provides a fixed-point construction which starts from a classical model for a base language without a truth predicate and provides a way to generate a model for a language containing such a predicate. One strategy for generating this kind of model is to use Kleene's strong valuation schema. According to this schema, negation is defined as 1 minus the value of the negated formula and disjunction is defined as the maximum of the values of the disjuncts (the other connectives are defined in terms of these two). Kripke's insight is that we can construct different strong Kleene models with the additional feature that the value assigned to any formula A is the same as the value assigned to the formula A *is true*. Any theory satisfying this property is usually considered to be a *naive truth theory*.

The three-valued logics K_3 and LP are notorious examples of this (for an overview of these logics see [2]). The only important (non-philosophical) difference between K_3 and LP has to do with the consequence relation defined by each of these logics. In the first one, an argument is valid if it preserves the value 1, while in the second one an argument is valid if it preserves the non-0 values. This difference has a major impact on the set of valid inferences and formulas. Crucially, both the Law of Excluded Middle and *Reductio ab Absurdum* fail in K_3 but not in LP. And both *Ex Contradictione Quolidbet* and Disjunctive Syllogism fail in LP but not in K_3.

However, they have an important feature in common. Consider a Liar sentence λ that is equivalent with its own negation. Both in K_3 and LP the sentence λ is categorized by means of the intermediate value $\frac{1}{2}$. This gives a consistent assignment to λ given that the value of $\neg\lambda$ is defined as 1 minus the value of λ.

In this talk I want to suggest that there is another strategy available to deal with the Liar sentence and to obtain a naive truth theory. The interesting aspect of this strategy is that it does not involve the introduction of a third semantic category or a modification of the notion of validity. The idea is to define negation using a non-deterministic matrix (see [1] for the idea of a non-deterministic matrix). This can be done in several ways. I will prove that however we do it, the resulting logic is compatible with a naive truth theory. Moreover, I will give two-valued versions of the theories K_3 and LP, and show how to give a Kripke-style definition of the truth predicate for these logics.

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Self-referential truths and falsities

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One of the most common diagnostics with regard to sentences that give rise to self-referential or infinitary semantic paradoxes is that their reference patterns are "pathological" (Herzberger 1970). Given the contradictions that arise from them, they are usually regarded as neither true nor false (or both true and false) and handled formally in non-classical logics such as K3 (or its paraconsistent dual, LP). The aim of this paper is to analize the consequences of introducing a "pathologicality" operator in the same language in which paradoxical sentences are formulated. This will imply the existence of new self-referential –and possibly pathological– sentences. Namely, δ_1 := The sentence δ_1 is pathological; and δ_2 := The sentence δ_2 is not pathological. A five-valued semantics will be given for the resulting language. I argue for its philosophical justification by discussing (i) that pathological sentences aren't necessarily neither true nor false, or both true and false; (ii) that some self-referential, i.e. pathological, sentences can be just true (or just false) precisely in virtue of being pathological.

Combining preferences and actions into a logic: The emergence of a general principle.

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In logic Von Wright?s work is commonly taken as a first hand reference of the study of preference judgments and their logical relations. Von Wright (1963) develops a preference axiom system and an implicit semantic that has some flavor of a kripke semantic. Although it is clearly outline that the change of preference will be left out of the inquiry it is acknowledge that the concept has nevertheless an intrinsic dynamic dimension. Moreover, within the theory of computer science Dynamic Logic is used to prove correctness properties of computer programs. This is no more than saying that a computer program is in fact nothing but a sequence of actions of a certain kind.

As preferences are exerted over states, we can say that actions act over these same states. It seems natural to think of many examples involving some interaction between preferences and actions, that is, preferences over actions or actions over preferences. Asking about what kinds of restrictions govern these interactions it amounts to study the ways in which actions and preferences can be mixed up. From the point of view of logic this kind of inquiries are developed on what is known as combining logics and it extends to mixing different modal operators (each one governed by its own logic) and merging them into a new logic.

The aim of this article is to propose a combination of logics of preference and actions to capture a qualitative concept of preference. In doing so, we will encounter with a general principle governing this interaction expressing that in either way that preference and actions are combined they don't disturb each other. As we can see from an agents point of view this is as general as it can get, the next step is to study what kind of repercussions the system has when the former principle is weakened.

The plan for this article is as follows. Firstly a short presentation of dynamic logic and the logic of preference is given. After that we will introduce the semantics for these systems. Once this is done we consider how to combine preferences and actions by means of fusion. Furthermore a general principle of interaction will be formulated and discussed. Finally we will formulate some forms of wakened the former principle and its effect over the system.

Minimum many-valued modal logic over residuated lattices via duality

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One of the latest and most challenging trends of research in non-classical logic is the attempt of enriching many-valued systems with modal operators. This allows us to formalize reasoning about vague or graded properties in those contexts (e.g., epistemic, normative, computational) that require the additional expressive power of modalities. This enterprise is thus potentially relevant not only to mathematical logic, but also to philosophical logic and computer science.

A very general method for introducing the (least) many-valued modal logic over a given finite residuated lattice is described in [1]. The logic is defined semantically by means of Kripke models that are many-valued in two different ways: the valuations as well as the accessibility relation (viewed as the associated characteristic function) among possible worlds are both many-valued ways, the valuations as well as the accessibility relation among possible worlds being both many-valued. The work in [1] also shows that providing complete axiomatizations for such logics, even if we enrich the propositional language with a truth-constant for every element of the given lattice, is a non-trivial problem, which has been only partially solved to date.

In this presentation we report on ongoing research in this direction, focusing on the contribution that the theory of natural dualites [2] can give to this enterprise. We show in particular that duality allows us to adapt the method used in [1] to prove completeness with respect to local modal consequence, obtaining completeness for global consequence, too (a problem that, in full generality, was left open in in [1]). Besides this, our study is also a contribution towards a better general understanding of quasivarieties of (modal) residuated lattices from a topological perspective.

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Propositions, Modality and Quantification

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This work is concerned with the translation of quantified modal logic embedded with a general semantic (hereafter referred to as *QML-G*) [2], [1], into a many-sorted language. Modal correspondence theory, which has been inspired by the work developed by Johan van Venthem in the 1980s, focuses on the relationship of modal logic with other logics, particularly with classical logic. It allows to exhibit the expressive capacity of modal language with respect to other languages and its accuracy compared to certain fragments of first and second order languages. Outstanding results in this area are, for example, the precise delimitation of the fragment of first order logic defined by modal logic. In a celebrated result, J. van Benthem showed that the set of modal formulas which are definable in the language of first order logic are exactly those formulas of the modal language, which remain invariant under bi-simulation. This correspondence between suitable structures for two languages, allows also to obtain some remarkable philosophical implications. Connecting two diverse languages, it permits to shed light on some potentials and limitations of these languages, which otherwise could have remained concealed. Extending modal correspondence theory to quantified modal logic proved to be much more difficult. In a recent paper, [3], Johan van Benthem also started dealing with the problem. In [3], some results are presented concerning the relationship between quantified modal language with a Kripkean semantics and many-sorted models, on which the modal language is interpreted. The expressive power of standard quantified modal language with kripke semantics is considered, assuming increasing domain inclusion for quantifiers. As a result, a minimal quantified modal logic is validated that fusions standard predicate logic with the basic propositional modal logic K. In order to determine the expressive power of this system, van Benthem draws on very known techniques of modal correspondence theory for propositional modal logic. After defining a translation for modal formulas into a two-sorted language L_{corr}, which has variables for objects and worlds, it turns out that -at a syntactic level- modal predicate logic can be seen as a fragment of the two-sorted language L_{corr}. Within this framework, its characteristic semantic invariance can be defined -according to van Benthem- as a mix of two structural relations between models, which are very well known: modal *bi–simulation* and the corresponding notion for first order languages, namely, the notion of *potential isomorphism.*

Our proposal in this paper, follows the method of Maria Manzano, [4], [5] in order to obtain a representation theorem between *QML-G* and Many-Sorted Logic. The referred procedure is as follows. The signature of the logic under consideration -let call it *XL*- is transformed into a many-sorted structure, by translating the expressions of *XL* to the many-sorted logic *MSL* and the very structures of *XL* are changed to many-sorted structures. Hence, it is necessary to define a recursive function TRANS for translation and also a direct conversion of structures, *CONV*. The direct conversion of structures gives as result the equivalence between validity of any formula φ in the former structures of the logic under study –let call it $\mathcal{ST}\,(XL)$– and the validity of the universal closure of its translations to many-sorted logic, i.e., $\forall\mathrm{TRANS}\,(\varphi)$, under the class $\mathcal{S}^*$ of structures obtained by conversion. That is, $\mathcal{S}^* = CONV(\mathcal{ST}\,(XL))$. Consequently, the key to obtain definitions for *TRANS* and *CONV* depends on the capacity to simplify the proof of the semantical equivalence. Additionally, the relevance of the results, which can be obtained, is bound to the eventual axiomatization of $\mathcal{S}^*$. In certain cases, it will suffice to axiomatize a broader class containing $\mathcal{S}^*$. It is important to observe that translations of formulas belonging to *QML-G* will not be true in *MSL* straightforward. Merely applying the conversion between structures *CONV*. What is required is an explicit representations of all sets which can be defined by using modal formulas in structures of the general semantics for *QML*. The most difficult part has to do with the semantic of quantifiers, where general semantic appeals to the notion of ”admissible propositions” and considerably departs from traditional Kripke semantics. Incidentally an alternative approach to propositions –based on an idea already advanced by Rich Thomason [6]– will be addressed, wherein propositions are taken as primitive elements and interpreted in the setting of the theory of types. Notably, the correspondence with the appropriate many-sorted structures, can also help to dilute some troubles concerning the interpretation of propositions as sets of possible worlds.

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Permutable bisimulation equivalences of Kripke frames

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One reason for the failure of Beth's definability theorem on (modal) logics is that there are new connectives "algebraically" definable. That is, in the variety of dual objects (Boolean algebras with operators, BAOs) there are *algebraic functions* that are not terms of the variety. Algebraic functions are those defined systems of equations having unique solutions in the variety.

Our research program consists in studying algebraic functions of varieties of BAOs, first in the case with discriminator. For finitely many finite algebras, algebraic functions are essentially characterized as those preserved by intersection of subalgebras. Via the duality, we have to consider the structure of all bisimulation equivalences of a Kripke frame with the operation of join.

In this work in progress, we started studying a very small case, that of Kripke frames that are finite linear (symmetric) graphs $\mathbf{L} = \langle n1, R\rangle$, where $x\ R\ y$ iff $|x - y| \leq 1$. Already in this case, the computation of the join is non trivial and shows a very interesting combinatorial behavior. Under the assumption of permutability we provide a formula; actually we believe that we have that all pairs of bisimulations having a join different from $(n1) \times (n1)$ permute.

A topological approach to Monadic Tetravalent Modal algebras

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Tetravalent modal algebras (or $\mathcal{TMA}$) were the the last algebraic structure introduced by Antonio Monteiro and they were studied by his last Ph.D student I. Loureiro (see [6]). Also, A.V. Figallo ([4]) and A. Ziliani ([8]) were interested in this subject. On the other hand, J. Font and M. Rius studied this class of algebras with the methods of *abstract algebraic logic* ([5]). More recently, S. Celani proved that the variety of $\mathcal{TMA}$ has the Amalgamation Property and the Superamalgamation. Besides, M. Coniglio and M. Figallo ([3]) studied them under the perspective of paraconsistent logics.

In our work we consider monadic tetravalent modal algebras (or $m\mathcal{TMA}$). We define them following Halmos' studies on monadic Boolean algebras. Here, we treat $\mathcal{TMA}$ as especial pseudocomplemented De Morgan algebras ([5]). Then, we could use their topological representation theory (Halmos-Priestley's duality) in a fruitful way. In order to study and determinate the generating algebras (finite and infinite) of the variety of $m\mathcal{TMA}$, we used concepts and properties of *general tolopogy* such that *convergence and accumulation* of nets (or Moore?-Smith sequences), and the *extension by continuity* theorem on T_3-spaces.

On the other hand, we consider the variety of $gm\mathcal{TMA}$ that contains as a proper sub-variety the variety of $m\mathcal{TMA}$. Besides, the variety of $gm\mathcal{TMA}$ is a proper sub-variety of the variety of monadic De Morgan algebras (or $m\mathcal{DMA}$) ([7]). Then, as a consecuence of our topogolical studies for $m\mathcal{TMA}$, we can prove that if we have a $\mathcal{TMA}$ A, and we equipped it with an existential quantifier $\exists$ (of $m\mathcal{DMA}$), then we have that $(A, \exists)$ is a simple algebra of $gm\mathcal{TMA}$ and also of $m\mathcal{DMA}$. This fact show that the lattice of subvarieties of $m\mathcal{DMA}$ is more complex than of that bonuded distributive lattices with a quantifier (Q-lattices) introduced by Cignoli.

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From Dyadic Semantics to Translations into Classical Logic

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Suszko's thesis, as it is nowadays known, states that there are only two logical values, *true* and *false*, and that any many-valued logic can be provided with a bivalent semantics (see [3]). In [1], it is pointed that Suszko's original presentation of this reductive result is quite non-constructive and it is provided a method to effectively construct a kind of bivalent semantics, named *gentzenian semantics*, for logics that have a truth-functional finite-valued semantics and a 'sufficiently expressive language'. Then, the *dyadic semantics* is defined as a specialization of gentzenian semantics where 'quasi tabular' decision procedures can be stablished, "in a deliberate intent to capture the computable class of such semantics". Moreover, it is shown that some logics that are no characterizable by truth-functional finite-valued semantics are characterizable by dyadic semantics, as it is the case of da Costa's paraconsistet logic C_1 (in [2], dyadic semantics for other paraconsistent logics are also defined). It is important to mention that *dyadic semantics* are generally non-truth-functional, but they are usefull, for instance, to define tableau decision procedures. Here, it is shown that dyadic semantics is also useful to obtain conservative translations of non-classical logics into the classical propositional logic (CPL).

Given a logic $\mathcal{L} = \langle For_{C_\mathcal{L}}, \Vdash_\mathcal{L} \rangle$ characterized by a dyadic semantics, where the set Sem is defined by a set of conditional clauses B, a method to obtain a conservative translation of $\mathcal{L}$ into CPL consists in the following steps:

1. For each formula δ in the consequent of a conditional clause in B construct a truth table, including a column for each formula φ_i^j for which the truth value of δ depends on (i.e. the formulas on the antecedent of the respective conditional clauses) and a column for a *hidden propositional variable* p_δ. Use the rows to write all the possible assignation of truth values (in $\{T, F\}$) to formulas φ_i^j and to p_δ, and set the respective values of δ in accordance with the conditional clauses in B. If in some row the value of δ is not uniquely determine by the values of formulas φ_i^j, set to δ the value of p_δ. In this way, it is obtained a bivalued truth-functional definition of δ, in terms of the truth values of formulas φ_i^j and p_δ, validating the conditions in B and representing the non-deterministic character of the truth value of δ by including the new propositional variable p_δ. This can be viewed as a method to recover the truth functionality of δ.
2. For each truth table constructed in the previous step for a formula $\delta \in For_{C_\mathcal{L}}$, obtain a CPL-formula $CPL(\delta)$ that represents the boolean values of δ in the truth table, in terms of formulas φ_i^j and p_δ. This is always possible since CPL is functionally complete, moreover there exists algorithmic methods to convert truth tables into CPL-formulas.
3. Define the set of classical propositional variables as $P_{CPL} = P \cup P'$, where P is the set of propositional variables over which $For_{C_\mathcal{L}}$ is generated and $P' = \{p_\delta \mid \delta \in For_{C_\mathcal{L}}\}$ is a set of *hidden variables*. Note that P_{CPL} is just a denumerable set of propositional variables, conveniently partitioned into two sets. Let $For_{C_{CPL}}$ be the set of formulas generated by the set of classical propositional connectives $C_{CPL} = \{\ominus, \owedge, \ovee, \circledarrow, \circledleftrightarrow\}$ over P_{CPL} (classical connectives are here circled in order to differentiate them from the connectives in $C_\mathcal{L}$). Then recursively define the translation function $T\colon For_{C_\mathcal{L}} \to For_{C_{CPL}}$ in the following way:
   (a) For propositional variables $p_i \in P$ set $T(p_i) = p_i$.
   (b) For each n-ary connective $c \in C_\mathcal{L}$, let $CPL(\delta^1), \ldots, CPL(\delta^m)$ be the formulas obtained in step 2 for formulas $\delta^k = c(\varphi_1^k, \ldots, \varphi_n^k)$. Replace formulas φ_i^j in $CPL(\delta^k)$ by $T(\varphi_i^j)$, for $1 \leq k \leq m$ and set:

$$T(c(\varphi_1, \ldots, \varphi_n)) = \begin{cases} CPL(\delta^1), & amp; \text{if } \varphi_i = \text{ satisfies schema } \varphi_i^1 \\ & amp; (\text{for each } 1 \leq i \leq n); \\ \vdots & \\ CPL(\delta^m), & amp; \text{if } \varphi_i = \text{ satisfies schema } \varphi_i^m \\ & amp; (\text{for each } 1 \leq i \leq n); \end{cases}$$

Theorem 2. *Given a logic* $\mathcal{L} = \langle For_{C_\mathcal{L}}, \Vdash_\mathcal{L} \rangle$ *characterized by a dyadic semantics, the translation function* $T\colon For_\mathcal{L} \to For_{C_{CPL}}$ *obtained by following the method described above is a conservative translation.*

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Symmetries in Modal Logics

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Symmetries have received much attention during the last years in the automated reasoning community, especially in propositional satisfiability solving (SAT solving) [1, 2, 4, 3, 5] but also in other logics [6, 7, 8], as they can help in solving many hard problems.

If we are able to recognize the symmetries of a problem, we can use them to direct a search algorithm to look for solutions only in non-symmetric parts of the search space, thus reducing the overall difficulty.

In general, we speak about symmetry (of an object) if under any kind of transformation at least one property of the object is left invariant. In automated reasoning, we can define a symmetry of a problem as a permutation of its variables (or literals) that preserves its structure (its syntactic form) and, hence, its set of solutions (its models).

In this work, we investigate symmetries in the context of modal logics. We show how permutations of propositional literals can be used to define symmetries for basic modal formulas in clausal form and how they share many similar properties with symmetries for the propositional case. We prove that symmetries of a basic modal formula partition the model space into equivalence classes such that each equivalence class contains only models satisfying the formula or models not satisfying it, i.e.,

Theorem: If σ is a symmetry of φ then $\mathcal{M} \in \mathsf{Mods}(\varphi)$ if and only if $\sigma(\mathcal{M}) \in \mathsf{Mods}(\varphi)$.

We also prove that symmetries can be used as an inference mechanism, and therefore, they can be used to strengthen existing reasoning mechanisms, i.e.,

Theorem: Let φ and ψ be modal formulas and let σ be a consistent symmetry of φ. Then $\varphi \models \psi$ if and only if $\varphi \models \sigma(\psi)$.

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Modular Sequent Calculi with Systems of Rules for Classical Modal Logics

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Since their inception (brought about by Montague and Scott, independently), neighborhood frames have become a standard semantic resource for logicians and philosophers wishing to work with modal logics weaker than those characterized by relational frames. However, while the domain of application of these so-called "classical" modal logics and their neighborhood semantics grows ever wider, and the model theory of neighborhood structures becomes better understood (e.g. [2]), there remains a noticeable absence in the development of the proof theory for these systems. This paper takes some first steps towards filling this void. In particular, we introduce a labelled sequent system for the minimal classical modal logic, as well as several extensions.

An attractive feature of Hilbert-style axiomatizations of modal logics is that they are *modular*: axioms can be added or subtracted in a component-wise fashion, generating a well-behaved lattice of logics with a common base. Maintaining this modularity is challenging when developing Gentzen systems for these logics. Importantly, our approach allows us to preserve this feature: specific frame conditions are each associated with a set of rules, and various logics can then be obtained via the addition of these sets to a common core governing the logical connectives and modal operators. Furthermore, we show that these rules can be added in a cumulative manner without requiring us to forgo cut elimination (or other desirable properties of sequent systems).

Because labelled systems have been developed primarily in the context of logics characterized by relational semantics, it is not immediately obvious how to adapt this method when one is working with structures described by a relatively rich set-theoretical language. Unlike in the relational case, where representing properties of binary relations is often more or less straightforward, it is unclear how one ought to represent, in the labelled system, the richer structural information present when dealing with neighborhood frames. We address this problem by making use of the fact that one can simulate a classical modal logic by way of a normal polymodal logic (for details, see [3] or [1]). This allows us straightforwardly to borrow much of the basic proof-theoretic apparatus already developed for normal modal logics. Furthermore, working in the translated language facilitates a clearer understanding of the set-theoretic frame conditions from a purely relational perspective. However, the translation only helps establish a starting point, and leaves open the important question of how best to achieve a uniform approach incorporating the intermediate logics between **E** and **K**. The majority of the paper is concerned with answering this question.

In order to do so, we make use of so-called *systems of rules*, which allow labelled systems to be developed for theories described by *generalized geometric formulas* (this has been introduced very recently in [4]). We show that the neighborhood frame conditions (understood relationally) for the intermediate logics correspond to instances of this type, and so can be characterized by these types of rule systems.

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