Titles and Abstracts for HIFW02

Arthur Apter

Normal Measures and Tall Cardinals

I will discuss the number of normal measures a non-$(\kappa + 2)$-strong tall cardinal κ can carry, paying particular attention to the cases where κ is either the least measurable cardinal or the least measurable limit of strong cardinals. This is joint work with James Cummings.

Omer Ben-Neria

"The possible Structure of the Mitchell Order"

The Mitchell order is a partial ordering on normal measures. The possible structure of the Mitchell order on the set of normal measures at a fixed cardinal κ was previously studied by Baldwin, Cummings, Friedman and Magidor, Mitchell, Thompson, and Witzany. The purpose of this talk is to outline two forcing methods for realizing various well founded orders of cardinality $\leq \kappa$ as the Mitchell order at κ:

1. A first is used to realize orders in a wide family of well-founded orders called tame orders and requires large cardinal assumptions weaker than $o(\kappa)=\kappa+$.
2. The second method assumes a large cardinal property slightly stronger than a sharp to a strong cardinal, and is used to realize every well founded order as the Mitchell order.

Piotr Borodulin-Nadzieja

Measures on non-separable space

Every separable compact space is a support of a measure. There are not so many examples of compact spaces supporting measures which are not separable. Usually such spaces are rather big topologically (as e.g. the Stone spaces of measure algebras). We investigate the existence of small non-separable spaces supporting measures. We show that under MA(\omega_1) there is a non-separable support of a measure which has small \pi-character, although such space does not exist in the standard Random model. Moreover, under Martin's Axiom there is a non-separable support of a measure which cannot be mapped continuously onto the cube of weight \omega_1. This represents work joint with Grzegorz Plebanek.

David Chodounsky

Mathias forcing and generic ultrafilters

I will analyze the Mathias--Prikry forcing $M(F)$ where F is a filter in ω, and derive a condition for reals which is sufficient for genericity in this poset. As an application, I will present a characterization of ultrafilters in $\mathcal P(\omega)/I$ generic over $L(\mathbb R)$ for F$_\sigma$ ideals I (assuming proper large cardinal hypothesis).

Todd Eisworth

On λ-properness

We look at the problem of generalizing properness to the context of λ-support iterations for uncountable regular~λ. We will discuss some recent work, but one of our main goals is to provide an accessible introduction to this area and some of its peculiarities.

Márton Elekes

Set-theoretical properties of Hausdorff measures

Hausdorff measures are the main objects of study in fractal geometry and geometric measure theory, and they have recently been playing an important role in harmonic analysis, percolation theory, dynamical systems, and many other fields. A set $H \subset \mathbb{R}$ is of \emph{r-dimensional Hausdorff measure zero} if for every $\varepsilon>0$ there are intervals $\{(a_n, b_n) \}_{n \in \omega}$ covering H such that $\sum_{n \in \omega} (b_n - a_n)^r < \varepsilon$.

The σ-ideal of sets of r-dimensional Hausdorff measure zero is denoted by $\mathcal{N}^r$. We discuss how the cardinal invariants of this ideal can be fit into the Cicho\'n Diagram, and also that which of the inequalities that show up are consistently sharp. As applications we answer questions by Fremlin and by Humke-Laczkovich.

Menachem Kojman

The arithmetic or density

The kappa-density of a cardinal mu is the least cardinality of a collections of kappa-subsets of mu with the property that every kappa-subset of mu contains one from the collection. The density function D(mu,kappa) enjoys more regularity properties than cardinal exponentiation. In the talk we shall survey some of the properties and ask whether certain statements about the asymptotic behavior of density are consistent with ZFC.

Anja Komatar

Structural Ramsey Theory

We will discuss structural Ramsey theory and give examples by considering Ramsey classes of shaped partial orders. Given structures A,B, we say that $$C\to (B)^A_k,$$ if for any partition of substructures of C isomorphic to A into k classes, there is a B with all substructures A in the same class. A class $\mathcal{K}$ is Ramsey if for any $A,B \in \mathcal{K}$, such a $C\in \mathcal{K}$ exists. We will show how the partite construction is used to prove a class of structures is Ramsey.

Borisa Kuzeljevic

Forcing with matrices of countable elementary submodels

We analyze the forcing notion $\mathcal P$ of finite matrices whose rows consists of isomorphic countable elementary submodels of a given structure of the form H_{θ}. We show that forcing with this poset naturally adds a Kurepa tree. Moreover, if $\mathcal P_c$ is a suborder of $\mathcal P$ containing only ontinuous matrices, then the constructed Kurepa tree is almost Souslin, i.e. the level set of any antichain in T is not stationary in ω_1. This is joint work with Stevo Todorcevic

Heike Mildenberger

Destroying and completing maximal centred systems.

Suppose $({\mathcal C}, \leq)$ is a maximal centred system that serves as the reservoir for the pure parts and the $\leq$-relation on the pure parts of a notion of forcing. By forcing we destroy the maximality of ${\mathcal C}$, however, we do not diagonalise ${\mathcal C}$. Is there a way to complete ${\mathcal C}$ in the extension to a maximal centred system? An affirmative answer is useful for increasing certain forcing orders.

We use Ramsey-theoretic properties of the system for evaluating the the forcing and in order to establish an extended system that again enjoys these Ramsey-theoretic properties. These computations with forcings form part of the project to construct a model with a simple $P_{\aleph_1}$-point and a simple $P_{\aleph_2}$-point.

Justin Moore

There may be no minimal non σ-scattered linear orders.

In 1971 Richard Laver verified a longstanding conjecture of Fraisse by showing that the class of countable linear orders are well quasi-ordered by embeddability. In fact he proved that the broader class of σ-scattered linear orders is well quasi-ordered. Around the same time, James Baumgartner proved that it is consistent that any two $\aleph_1$-dense suborders or $\mathbb{R}$ are isomorphic and in particular that it is consistent that there is a minimal real type. It is natural to ask whether it is consistent that Laver's result is sharp: is it consistent that there are no linear orders which are minimal with respect to being non σ-scattered? We have established that this is fact the case. This is joint work with Hossein Lamei Ramandi.

Itay Neeman

Iterating $\aleph_1$-strongly proper posets with exact residue functions

We use countable side conditions to iterate certain $\aleph_1$-strongly proper orcing notions. In its most direct form the method applies to posets which are $\aleph_1$-strongly proper with residue functions of particular form that we call exact. Such residue functions exist for example when the quotients involved with the strong properness are countably closed. We describe the method in these situations.

In a more involved form the method is part of a proof of the consistency of Baumgartner's isomorphism principle at $\aleph_2$, that is the statement that any two $\aleph_2$-dense subsets of ${\mathbb R}$ are order isomorphic. We briefly discuss the connection.

Grzegorz Plebanek

Grzegorz Plebanek
(Mathematical Institute, University of Wrocław)

Orthogonal ideals on ω with applications to Banach spaces

joint work with A. Avilés and J. Rodríguez (University of Murcia)

Abstract. If $\mathcal{A}$ is a family of subsets of ω then the orthogonal ideal $\mathcal{A}^\perp$ is the family of all $B \subseteq \omega$ such that $A \cap B$ is finite for every $A \in \mathcal{A}$. Building on results of Todorcevic and Feng, we show that under the axiom of analytic determinacy there is a finite number of Tukey types of $\mathcal{A}^\perp$ for analytic families $\mathcal{A}$.

Given a Banach space X, let $\mathcal{K}(X)$ be the family of weakly compact subsets of X. The above theorem enables us to give a classification of separable Banach spaces X with respect to the lattice structure of $\mathcal{K}(X)$. We also analyse $\mathcal{K}(X)$ equipped with the relations $\leq_\varepsilon$ of 'almost inclusion', where for $K, L \in \mathcal{K}(X)$, $K \leq_\varepsilon L$ means that $K \subseteq L + \varepsilon \cdot B_X$ (where B_X is the unit ball in X).

Daniel Soukup

Problems in graph theory and forcing

The aim of this talk is to give an overview of mostly open problems in combinatorial set theory concerning infinite graphs. We survey topics where forcing played an important role in finding (complete or partial) solutions and gave inspiration for further advances in ZFC. Classical themes include the investigation of the structural behaviour of graphs with large chromatic number and Ramsey/anti-Ramsey properties of graphs. Our hope is to leave the audience with a healthy list of easily accessible yet interesting open problems of varying difficulty.

Spencer Unger

The tree property at aleph_{omega^2+1} and aleph_{omega^2+2}

An old question of Magidor asks whether it is consistent that the tree property holds at every regular cardinal greater than aleph_1. Motivated by this question and building on recent work of Sinapova, we prove that it is consistent that aleph_{omega^2+1} and aleph_{omega^2+2} have the tree property when aleph_{omega^2} is a strong limit cardinal. This is joint work with Dima Sinapova.

Matteo Viale

Category forcing axioms and generic absoluteness for third order arithmetic

In this talk we will survey several results pinpointing that there are natural strengthening of forcing axioms which lead to generic absoluteness results for the theory of H(\omega_2). In particular these type of results give an a posteriori explanation of the success met by forcing axioms in settling problems formalizable in third order number theory. We will focus on two type of forcing axioms: the iterated resurrection axioms and the category forcing axioms. These axioms are naturally formulated in a set theory with classes and have the following general form:

Let \Gamma be a ``nice'' class of forcings closed under two steps iterations (proper, semiproper, SSP, CCC, Axiom A, etc....). Say that P\geq_{\Gamma} Q if there is a complete embedding of P\restriction P into Q for some p\in P.

Then all these forcing axioms can be formualted as the assertion that a certain class D\subseteq \Gamma is dense in the class partial order (\Gamma,\leq_\Gamma).

By varying D and \Gamma we can produce a variety of interesting forcing axioms yielding various forms of generic absoluteness results.

Part of these results are joint work on one side with Giorgio Audrito and on the other side with David Aspero.

Teruyuki Yorioka

Todorcevic's fragments of Martin's Axiom and its restrictions

Stevo Todorcevic introduced Ramsey theoretic statements which are implied by Martin's Axiom in 1980s. For example, for each n > 1, one of such statements is denoted by K_n which is the statement that every ccc forcing has property K_n. Note that MA_{aleph_1} implies all the K_n, and K_{n+1} implies K_n. There are several open questions about K_n's.
For example, it is unknown whether some K_n implies MA_{aleph_1},
and whether K_n implies K_{n+1}. I will talk old and new results on this research.