



## **New directions in the higher infinite**

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International Centre for Mathematical Sciences, Edinburgh

# **Abstracts and research statements**

[www.icms.org.uk/workshops/higherinfinite](http://www.icms.org.uk/workshops/higherinfinite)

## Welcome!

Welcome to the 2017 Young Set Theory workshop, *New directions in the higher infinite*. We are pleased to be able to host a Young Set Theory workshop in the UK for the first time this year.

For your convenience, this booklet contains a timetable, titles and abstracts for the talks, and titles for the posters. However, the main purpose of the booklet is to share the research statements of all of the participants. One of the major strengths of the Young Set Theory workshop series is the way it brings the participants together to talk about set theory, and these research statements help to facilitate this.

If you have any questions or concerns, please don't hesitate to ask one of us. Enjoy the conference!

— The organisers.

## Tutorials

### Large cardinals as reflection principles: a general framework

*Joan Bagaria*

With the introduction by Levy, Keisler, Scott, et. al. of model-theoretic ideas in the theory of large cardinals during the 60's and 70's of last century, it became progressively apparent that some of the most common large cardinal notions, such as inaccessibility, weak compactness, or measurability, could be characterized in terms of Reflection properties of the set-theoretic universe. Further results of Magidor, Solovay, Reinhardt, Silver, et. al. provided additional evidence that Reflection was indeed an underlying common feature of most large cardinal principles. In this tutorial I will present some recent results that yield new characterizations of many large cardinals in terms of what we call Structural Reflection (SR). Thus, SR provides a general framework for the study of the large cardinal hierarchy, yielding new insight into its hidden regularities.

### Anticlassification results in ergodic theory

*Matthew Foreman*

“Impossibility Results” are well known in mathematics: the circle cannot be squared with a ruler and compass, the quintic is unsolvable using radicals, the word problem in group theory is unsolvable and many others. Descriptive Set Theory provides tools for proving such impossibility problems in analysis.

Borel sets in a Polish space generously encode all tests/procedures/transfinite protocols that can be carried out using arbitrary amounts of countable information. For example, saying that an equivalence relation  $E$  is not Borel can be interpreted as saying that there is no way of determining whether  $x E y$  without using uncountable information such as the Axiom of Choice.

In 1932 von Neumann proposed classifying measure preserving diffeomorphisms of compact manifolds up to the equivalence relation of measure isomorphism. These talks give an application of Descriptive Set Theory showing that this equivalence relation is not Borel. It also places the equivalence relation in the quasi-ordering of analytic equivalence relations up to Borel reducibility.

The proof requires an adaptation of the method of Approximation by Conjugacy, and thus yields new results in ergodic theory itself related to the

celebrated Realization Problem.

## **Gentle introduction to descriptive inner model theory**

*Grigor Sargsyan*

Assuming participants have zero knowledge of inner model theory, we will present some of the motivations and goals of inner model program. Then we will introduce a few modern techniques such as generality iterations, core model induction, hod analysis and etc.

## **Almost disjoint and eventually different families**

*Asger Törnquist*

There has recently been a lot of new results surrounding the study of definable almost disjoint families of subsets of the natural numbers, and of eventually different families of functions on the natural numbers.

The starting point for all of this is an old result, dating back to 1969, by Adrian Mathias that says that no infinite maximal almost disjoint family can be analytic (in the sense of descriptive set theory). In 2014 I proved that in Solovay's model there are no infinite maximal almost disjoint (mad) families. This result was improved last year by Shelah and Horowitz, who showed that a model of ZF with no infinite mad families can be obtained without using an inaccessible. Shortly after, Shelah and Horowitz solved another long-standing and related problem, by showing that there *is* a Borel maximal eventually different family. In another interesting development, Neeman and Norwood showed that under  $AD^+$  there are no mad families.

In this tutorial, I will give an overview of these many new results about almost disjoint families and eventually different families. The two first lectures will be dedicated to discussing the results about mad families due to myself, Shelah, and Horowitz, and I will present an elegant, short proof due to David Schritterser that there is a closed maximal eventually different family. The last two lectures I will present some new work, joint with Schritterser and Haga, in which we consider almost disjoint families relative to other ideals than the ideal of finite sets, and also consider mad families in the context of determinacy assumptions.

## Invited Talks

### Non-standard forcing types

*Carolin Antos-Kuby*

The standard version of forcing usually denotes forcing with a set of conditions in ZFC. Here we want to have a look at forcing outside of this particular framework by varying the size of the forcing notion and the axiomatization in which forcing takes place. We will give an introduction to the different ways in which class forcing can be approached in different backgrounds such as variants of ZF(C) and Morse Kelley Class Theory (MK) and the problems connected to verifying the Definability and Truth Lemma in these contexts. Furthermore we present a way of dealing with hyperclass forcing, a forcing notion containing class-sized conditions, and show how it can be made usable to solve questions like forcing minimal models of a variant of MK.

### Generic absoluteness for Chang models

*David Asperó*

The main focus of the talk will be on extensions of Woodin's classical result that, in the presence of a proper class of Woodin cardinals,  $\mathcal{C}_\omega^V$  and  $\mathcal{C}_\omega^{V^P}$  are elementarily equivalent for every set-forcing  $P$  (where  $\mathcal{C}_\kappa$  denotes the  $\kappa$ -Chang model).

1. In the first part of the talk I will present joint work with Asaf Karagila in which we derive generic absoluteness for  $\mathcal{C}_\omega$  over the base theory ZF+DC.

2. Matteo Viale has defined a strengthening  $\text{MM}^{+++}$  of Martins Maximum which, in the presence of a proper class of sufficiently strong large cardinals, completely decides the theory of  $\mathcal{C}_{\omega_1}$  modulo forcing in the class  $\Gamma$  of set-forcing notions preserving stationary subsets of  $\omega_1$ , i.e., if  $\text{MM}^{+++}$  holds,  $P \in \Gamma$ , and  $P$  forces  $\text{MM}^{+++}$ , then  $\mathcal{C}_{\omega_1}^V$  and  $\mathcal{C}_{\omega_1}^{V^P}$  are elementarily equivalent.  $\text{MM}^{+++}$  is the first example of a “category forcing axiom.”

In the second part of the talk I will present some recent joint work with Viale in which we extend his machinery to deal with other classes  $\Gamma$  of forcing notions, thereby proving the existence of several mutually incompatible category forcing axioms, each one of which is complete for the theory of  $\mathcal{C}_{\omega_1}$ , in the appropriate sense, modulo forcing in  $\Gamma$ .

## **Combinatorics and forcing at the successor of a singular cardinal**

*Mirna Džamonja*

In our work on universal graphs at the successor of a singular cardinal (several papers with collaborators including Cummings, Komjath, Magidor, Morgan and Shelah) we have developed a technique of forcing combinatorial results at such a cardinal using a mixture of a large cardinal forcing, iteration techniques and Prikry-type forcing. The technique had developed into a stage where it can be called a framework, but not really an axiom: that is, we have an idea of what kind of forcing it can be applied to, but we do not have a characterisation. I will discuss challenges in this programme and some new ideas under consideration.

## **Playing with equivalent forms of CH**

*Silvia Steila*

There are several mathematical statements which are equivalent to the Continuum Hypothesis (CH). For instance a famous equivalence by Sierpinski is: CH holds if and only if there are two subsets  $A$  and  $B$  of the real plane, whose union covers the real plane and such that any vertical section of  $A$  and any horizontal section of  $B$  are countable. In this talk we are going to analyse some variants of such statements and relate the corresponding versions to opportune relations between cardinal invariants. (On-going work with Alessandro Andretta and Raphael Carroy.)

## **Cardinal characteristics and partition relations**

*Thilo Weinert*

For quite some time, partition relations were considered assuming the generalized continuum hypothesis. After forcing entered the stage, independence results were pursued as well. Even later, connections between them, cardinal characteristics and other combinatorial principles started to be considered by Joji Takahashi, Stevo Todorcevic and Jean Larson. I will give an account of the history of these endeavours and recent advances made in collaboration with William Chen and Chris Lambie-Hanson. Towards the end I am going to give an outlook towards possibilities of future research.

## The perfect set property for universally Baire sets of reals

*Trevor Wilson*

A set of reals is said to have the perfect set property (PSP) if it is countable or it has a perfect subset. Suslin showed that all analytic sets have PSP. The PSP for more complex sets of reals is independent of ZFC. In particular, Gödel and Solovay showed that PSP for coanalytic sets and PSP for sets of reals in  $L(\mathbb{R})$  are both equiconsistent with the existence of an inaccessible cardinal. The PSP for universally Baire (uB) sets of reals, which generalize the coanalytic sets, is also known to be independent of ZFC. We show that PSP for universally Baire sets and PSP for sets of reals in  $L(\mathbb{R}, \text{uB})$  are both equiconsistent with the existence of a large cardinal that we call “Shelah for remarkability.” Such cardinals resemble Shelah cardinals but are much weaker and can exist in  $L$ . This is the result of joint work with Ralf Schindler.

## **Posters**

### **The tree property with a large gap**

*Radek Honzik and Sarka Stejskalova*

### **Incompatible actions of locally compact group**

*Manuel Inselmann*

### **Countable Support Products of Creature Forcings**

*Lukas Daniel Klausner*

### **Erdos-Ulam ideals vs. simple density ideals**

*Adam Kwela*

### **Ideals of nowhere dense sets in some topologies on integers**

*Marta Kwela*

### **The generalised shift graph**

*Milette Riis*

### **Magic sets**

*Salome Schumacher*

### **Introduction to Haar-small sets**

*Jarostaw Swaczyna*

### **Chains of $P$ -points**

*Jonathan Verner*

## Registered Participants

**1 Shaun Allison** (Carnegie Mellon University)

**2 Carolyn Barker** (University of Leeds)

I am a second year PhD student at the University of Leeds supervised by Professor John Truss. My work has focussed on partial orders and homogeneous structures.

Initially I worked on a construction of uncountable partial orders that exploited stationary sets to obtain certain weak homogeneity properties that would not be possible in the countable case, but this ran into technical difficulties. I also studied automorphisms of homogeneous structures that are generic in the sense of Truss, and investigated the existence and properties of mutually generic automorphisms (that is, pairs of automorphisms whose conjugacy class is comeagre in the product topology).

More recently I have been investigating some questions arising from my supervisor's work on Ehrenfeucht-Fraïssé games on coloured linear orders. As well as studying in the original context of linear orders I am also trying to generalise some results and ideas to the wider class of partial orders. Naturally there are many more possibilities for structures and so part of the problem is finding the most appropriate way of subdividing the possibility space. Restricting to trees for example heavily constrains the ways in which the  $n$ th move can be related to the  $(1, \dots, (n - 1))$ th moves, and so there are far fewer  $n$ -equivalence classes than when we consider all partial orders.

**3 Gianluca Basso** (Université de Lausanne and Università di Torino)

Supervisors: *Riccardo Camerlo and Jacques Duparc*

My research is focused on the projective Fraïssé limits of finite topological structures of some language  $\mathcal{L}$  and the compact metric spaces that are obtained as their quotients. The concept of projective Fraïssé limit was introduced by T. Irwin and S. Solecki in [5]. Key results in this area were obtained by R. Camerlo in [4], who characterized the quotients of the projective Fraïssé limits of finite graphs and by A. Kwiatkowska in [6]. More recent developments include [1] and [2] by D. Bartošová and A. Kwiatkowska, [7] by A. Panagiotopoulos and [3] by myself and R. Camerlo.

The results have interesting applications in Continuum Theory and Topological Dynamics of Groups of Automorphisms. In particular, D. Bartošová and A. Kwiatkowska in [2] have extended the Kechris-Pestov-Todorčević

correspondence, which links Ramsey Theory, Fraïssé Theory and Topological Dynamics, to the dual context of projective Fraïssé Theory. Recent work with R. Camerlo aims to advance the research in such direction.

## References

- [1] D. Bartošová, A. Kwiatkowska, *Lelek fan from a projective Fraïssé limit*, *Fundamenta Mathematicae* 231 (2015), 57–79.
- [2] D. Bartošová, A. Kwiatkowska, *Gowers’ Ramsey theorem with multiple operations and dynamics of the homeomorphism group of the Lelek fan*, *J. Combin. Theory Ser. A* 150 (2017), 108–136.
- [3] G. Basso, R. Camerlo, *Arcs, hypercubes, and graphs as quotients of projective Fraïssé limits*, *Math. Slovaca*, (to appear).
- [4] R. Camerlo, *Characterising quotients of projective Fraïssé limits*, *Topology and its Applications* 157 (2010), 1980–1989.
- [5] T. Irwin, S. Solecki, *Projective Fraïssé limits and the pseudo-arc*, *Transactions of the American Mathematical Society* 358 (2006), 3077–3096.
- [6] A. Kwiatkowska, *Large conjugacy classes, projective Fraïssé limits and the pseudo-arc*, *Israel Journal of Mathematics* 201 (2014), 85–97.
- [7] A. Panagiotopoulos, *Compact spaces as quotients of projective Fraïssé limits*, *ArXiv e-prints* (2016), available at 1601.04392.

## 4 Thomas Baumhauer (Technische Universität Wien)

A cardinal characteristic is the minimal size of a set with certain properties. Some of the most important cardinal characteristics are associated with the ideal  $\mathcal{M}$  of all meager subsets of  $2^\omega$  (or  $\omega^\omega$ ) and the ideal  $\mathcal{N}$  of all sets of Lebesgue measure zero. Their cardinal characteristics are related to each other in Cichoń’s famous diagram. Recently there has been increasing interest in analogous cardinal characteristics on  $2^\kappa$  (or  $\kappa^\kappa$ ) for uncountable  $\kappa$ . While there exists a straightforward generalization of the meager ideal seemingly there is none for the null ideal. However recently Saharon Shelah came up with a promising candidate in the case that  $\kappa$  is inaccessible. In my Ph.D. thesis (supervised by Martin Goldstern at the Technical University of Vienna) I plan to investigate Cichoń’s diagram for this generalized case, in particular proving independence results using forcing iterations.

## 5 Hazel Brickhill (University of Bristol)

I am finishing my PhD in set theory at the University of Bristol, supervised by Professor Philip Welch. The focus of my PhD research has been on the notions of generalised stationarity introduced in [1] and the complimentary generalised club sets. This is related to the widely studied phenomenon of stationary reflection. In particular I have worked on formulating combinatorial principles related to  $\diamond$  and  $\square$  principles using generalised clubs and stationary sets and how these principles relate to small large cardinal notions such as indescribability and ineffability. I have also investigated how these sets occur in the constructible universe  $L$ , and I want to look further at whether what can be said about  $L$  generalises to larger inner models. I am also interested in how large cardinal notions such as supercompactness relate to my generalised  $\square$  principles.

## References

- [1] Bagaria, J., Magidor, M., and Sakai, H. (2015) *Reflection and indescribability in the constructible universe*. Israel Journal of Mathematics 208.1: 1-11.

## 6 Andrew Brooke-Taylor (University of Leeds)

Large cardinals lie at the centre of my interests in set theory. I have worked on their connection with forcing, looking particularly at preservation of large cardinal properties while doing various forcings. More recently I've looked at the analogues of cardinal characteristics of the continuum for cardinals  $\kappa$  greater than  $\omega$ . In some cases to even make sense of the definitions  $\kappa$  needs to be inaccessible, and in many cases in order to transfer known results from the  $\omega$  case up to  $\kappa$  one seems to need  $\kappa$  to have a large cardinal property such as weak compactness or even supercompactness.

I'm also very interested in applications of set theory to other parts of mathematics, particularly by way of category theory. For decades the theory of locally presentable and accessible categories has been developed using large cardinal axioms such as measurable cardinals, strongly compact cardinals, and Vopěnka's Principle. This is a well-connected area of mathematics: in 2005 a paper of Casacuberta, Scevenels and Smith used a result from this area to show that Vopěnka's Principle resolves a long-standing open problem in algebraic topology (the large cardinal required for the proof has since been reduced by Bagaria, Casacuberta, Mathias and Rosický), and in a dif-

ferent vein, accessible categories have recently been shown to be intimately connected with AECs in model theory. I have results with Joan Bagaria and Jiří Rosický bringing a set-theoretic perspective to bear to improve results in this area; my work with Rosický in particular reduced the large cardinal required for a proof of the statement “every AEC is tame” to the optimal value.

## 7 Filippo Calderoni (University of Turin)

I’m a third year PhD student at the University of Turin, under the supervision of Luca Motto Ros, and I am expected to graduate in 2018. My research interests lie in Descriptive Set Theory and its applications to other areas of Mathematics. More specifically, I have been working in the theory of  $\Sigma_1^1$  quasi-orders. Given two quasi-orders  $P$  and  $Q$  defined over the standard Borel spaces  $X$  and  $Y$ , respectively, we say that  $P$  is *Borel reducible* to  $Q$  if and only if there is a Borel map  $f: X \rightarrow Y$  such that for all  $x, y \in X$  we have  $x P y \iff f(x) Q f(y)$ .

My research continues the work of Louveau and Rosendal, who found several concrete examples of *complete  $\Sigma_1^1$  quasi-orders*, that is, maxima in the class of  $\Sigma_1^1$  quasi-orders up to reducibility. I have been focusing on that notion of completeness and a strengthening of it, known in the literature as invariantly universality introduced by Camerlo, Marcone, and Motto Ros. The phenomenon of invariant universality is quite widespread in mathematics as most of the examples of complete  $\Sigma_1^1$  quasi-order turned out to be invariantly universal: for instance we prove invariant universality for the relation of embeddability on several spaces of countable structures. Examples include groups in [3], quandles, and fields of characteristic 0 in [1].

Further, working in the framework of Borel reducibility between relations over standard Borel  $\kappa$ -spaces, I proved that embeddability on torsion-free abelian groups of uncountable size  $\kappa = \kappa^{<\kappa}$  is a complete  $\Sigma_1^1$  quasi-order in the sense of Generalized Descriptive set Theory ([2]).

## References

- [1] Andrew Brooke-Taylor, Filippo Calderoni, and Sheila Miller. Invariant universality for quandles and fields. In preparation.
- [2] Filippo Calderoni. The complexity of embeddability between torsion-free abelian groups of uncountable size. Preprint.

- [3] Filippo Calderoni and Luca Motto Ros. Universality of group embeddability. Preprint.

## 8 Fabiana Castiblanco (Universität Münster)

I am a fourth year PhD student at the Universität Münster working under the supervision of Prof. Ralf Schindler.

Currently my research interests lie in Forcing and the interaction between Inner Model theory and Descriptive Set Theory. One example of the last point could be depicted when considering thin equivalence relations over the set of real numbers.

We say that  $E \subset \mathbb{R} \times \mathbb{R}$  is *thin* if there is no a perfect subset of  $E$ -inequivalent reals. In [4, Theorem 4.1.16.], Schlicht characterizes the inner models which have representatives in all equivalence classes of thin equivalence relations in the projective pointclasses of the form  $\Pi_{2n}^1$ . Connected to these results, in joint work with Philipp Schlicht, we proved that various arboreal forcing notions such as Sacks, Silver, Mathias, Miller and Laver forcing preserve sharps for reals. Furthermore, we showed that these forcing notions preserve also the existence of  $M_n^\#(x)$  for every  $n < \omega$ ,  $x \in {}^\omega\omega$  so all these forcings preserve  $\Pi_n^1$ -Determinacy. Also, we proved that those forcing notions does not add any new equivalence class to thin provably  $\Delta_{n+3}^1$  equivalence relations under the existence of  $M_n^\#(x)$ ,  $x \in {}^\omega\omega$ . As a corollary, we have that under the existence of sharps for reals Sacks, Silver, Mathias, Laver and Miller forcing do not change the value of the second uniform indiscernible  $u_2 = \sup\{(\omega_1)^{+L[x]} : x \in {}^\omega\omega\}$ , giving an partial answer to [2, Question 7.4].

Related to the lifting of elementary embeddings in generic extensions and therefore to the preservation of sharps for reals, in joint work with Ralf Schindler we characterize the reals  $x \in V$  that are generic by set forcing over  $L$ :

**Theorem.** *The following are equivalent:*

1.  $x$  is set-generic over  $L$
2. there is some  $p \in L$  such that for all elementary embedding  $j : L_\alpha \rightarrow L_\beta$  with critical point  $\kappa$ , where  $j \in L$ ,  $L_\alpha \models \text{ZFC}^-$  and  $p \in L_\kappa(\subset L_\alpha)$ , there is some  $\tilde{j} : L_\alpha[x] \rightarrow L_\beta[x]$  with  $\tilde{j} \supset j$  and  $L_\alpha[x] \models \text{ZFC}^-$ .

During the last months, we have focussed in the construction of models containing special sets of reals without a well-ordering of the reals as in the papers [1, 3]. In joint work with Ralf Schindler, we have constructed a model in which Lusin and Sierpiński sets coexist without a well-ordering of the reals. We aim to construct in such a model a Hamel basis.

## References

1. M. Beriashvili, R. Schindler, L. Wu, L. Yu. Hammel basis and well-ordering the continuum. Available at <http://ivv5hpp.uni-muenster.de/u/rds/index.html>.
2. D. Ikegami. Forcing absoluteness and regularity properties. *Ann. Pure App. Logic*, 161: 879-894, 2010.
3. R. Schindler, L. Wu, L. Yu. Hamel bases and the principle of dependent choice. Available at <http://ivv5hpp.uni-muenster.de/u/rds/index.html>.
4. P. Schlicht. *Thin Equivalence Relations in  $L(\mathbb{R})$* . PhD. Thesis, Westfälischen Wilhelms-Universität Münster, 2008.

## 9 Bill Chen (Ben-Gurion University of the Negev)

Generally, I am interested in combinatorial set theory, particularly at singular cardinals.

At Ben-Gurion University, I have been interested in the density function,  $D(\lambda, \kappa_0, \kappa_1)$ , defined as the minimum size of a family  $\mathcal{F}$  of subsets of  $\lambda$  of size  $\kappa_1$  so that for any subset  $Y$  of  $\lambda$  of size  $\kappa_0$ , there is some  $X \in \mathcal{F}$ . A well-known case is the stick,  $\uparrow := D(\omega_1, \omega_1, \omega)$ . Kojman proved that the density function satisfies a version of Silver's Theorem at singular cardinals, but much is still open in this direction.

In work with Shimon Garti and Thilo Weinert, we discovered that cardinal invariant statements, including those involving density numbers, can be used to replace instances of GCH to prove some classical negative partition relations.

Presently, I am also developing an interest in problems in set-theoretic topology relating to singular cardinals, particularly questions involving linearly Lindelöf spaces.

I have also worked on the theory of the mutual and tight stationarity properties of Foreman and Magidor. My papers are available at my personal website, <https://www.math.bgu.ac.il/~williams/>.

## 10 Ben De Bondt (Ghent University)

I am a student and will start in October my second year of a two-year master's program in pure mathematics at Ghent University. During this year, I will be working on a master's thesis, on a topic yet to be decided.

I have no personal research experience yet, but I am strongly interested in set theory and mathematical logic. I took various courses in these domains (mathematical logic, proof theory, set theory) and wrote my bachelor's thesis on the subject of the smaller large cardinals. My study of mathematical logic at Ghent university has been supervised by Professor Andreas Weiermann.

I think attending the workshop will be a great opportunity to get in touch with many different directions modern set theory is currently heading and to meet with the researchers working on these topics.

## 11 Stamatis Dimopoulos (University of Bristol)

I am a third year PhD student at the University of Bristol in the U.K., working under the supervision of Dr. Andrew Brooke-Taylor.

The general area in which I am working is the interaction between large cardinals and forcing. More specifically, I have looked at Woodin-like cardinals and in particular on a new notion called *Woodin for strong compactness*. I have shown that the first Woodin for strong compactness cardinal can consistently be the first Woodin or the first Vopěnka cardinal, a result which resembles Magidor's corresponding theorem about the first strongly compact cardinal ([1]). Such phenomena are described by the term *identity crisis* and my result is a new sign that identity crisis can be an intrinsic feature of compactness in the large cardinal hierarchy. My goal is to look at other forms of compactness and investigate whether this is the case.

### References

1. Magidor, Menachem. How large is the first strongly compact cardinal? or A study on identity crises. *Ann. Math. Logic* 10 (1):33-57, 1976.

## 12 Ben Erlebach (University of Cambridge)

## 13 Takehiko Gappo (University of Tokyo)

I am a fourth year undergraduate student at the University of Tokyo. There is no expert of set theory in my university, so I am studying under the supervision of Prof. Daisuke Ikegami at Tokyo Denki University and Prof. Toshimichi Usuba at Waseda University since last year.

I am interested in descriptive set theory and inner model theory. I learned basic fine structure theory in [1] and classical results of determinacy last year. For example, the covering lemma in  $L$ , projective determinacy, and AD in  $L(\mathbb{R})$ . Also, I started to learn inner model theory described in [2] and [3] from this April.

[1] Ralf Schindler and Martin Zeman, *Fine Structure*, in *Handbook of Set Theory*. Springer, 2010.

[2] William J. Mitchell and John R. Steel, *Fine Structure and Iteration Trees*. Springer-Verlag, 1994.

[3] John R. Steel, *An Outline of Inner Model Theory*, in Handbook of Set Theory. Springer, 2010.

## 14 Elliot Glazer (Harvard University)

I am an entering graduate student at Harvard University. Naturally, I'm not fully sure what exactly my research will be, but my advisor will almost certainly Professor Hugh Woodin, so I expect to do some sort of project related to inner model theory and determinacy. Depending on the state of the Ultimate L program within the next year, I might begin investigating the models Woodin has developed.

As an undergraduate, I mainly learned the basics of inner model theory and determinacy. My undergraduate thesis was some expository notes on  $0^\#$ , comparing the modern inner model theoretic approach to  $0^\#$  where it is defined as a baby mouse, to the approach based on Ehrenfeucht-Mostowski models. It is meant to introduce the reader to the fundamental ideas of inner model theory and how canonical inner models can be constructed from mice.

### References

1. Itay Neeman. Determinacy and Large Cardinals. *Proceedings of the International Congress of Mathematicians, 2*: 27–43, European Math. Society Publishing House, 2007.
2. Itay Neeman. Determinacy in  $L(\mathbf{R})$ . In Matthew Foreman and Akihiro Kanamori, editors, *Handbook of Set Theory*, pages 1877–1950. Springer Netherlands.
3. Ralf Schindler. *Set Theory: Exploring Independence and Truth*. Springer International Publishing Switzerland, 332 pp. 2014.
4. Hugh Woodin. *The 19th Midrasha Mathematicae Lectures*. 2016.

## 15 Fiorella Guichardaz (University of Freiburg)

As a PhD student in Freiburg, supervised by Prof. Mildenerger, I had the chance to learn some basics about different topics including set theory of the real line, constructibility, cardinal invariants, forcing technique with side conditions, equivalent definitions of proper forcing, an introduction of matrix iterations. I am now mostly focusing on forcing iterations with ord-transitive models, their constructions and the related properties.

A model  $M$  is called ord-transitive if for every  $x \in M$  which is not an ordinal,  $x \subseteq M$ . For those models some easy definitions, such as being a subset, are not more absolute. Ord-transitive models were introduced in [1] and [2] and used in [3] together with almost countable support iterations to prove the consistency of the Borel Conjecture and the dual Borel Conjecture.

### References

1. Saharon Shelah. Properness without elementarity, 2004.
2. Jakob Kellner. Non elementary proper forcing, 2012.
3. Martin Goldstern, Jakob Kellner, Saharon Shelah and Wolfgang Wohofsky. Borel Conjecture and dual Borel Conjecture, 2014.

## 16 Gabriele Gullà (University of Roma-Tor Vergata)

I am currently working, under the supervision of Dr. Paolo Lipparini, on some connections between topological properties (typically “generalized” compactness) of totally ordered spaces and some notion of large cardinals, i.e. compact and weakly compact.

See as references:

- 1) P.Lipparini, “More on Regular and Decomposable Ultrafilters in ZFC”;
- 2) P.Lipparini, “Ultrafilter Convergence in Ordered Topological Spaces”;

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My other themes of research and interests are:

- a) Set Theory: the general Theory of Ultrafilters, Large Cardinals,  $\Omega$ -Logic <sup>(1)</sup>, Forcing Axioms;
- b) Abstract Model Theory;
- c) Modal Logic and its connection with Set Theory <sup>(2)</sup>;
- d) Philosophical Aspects: Tarski/Quine/Kripke’s Theories, Mereology/Mereotopology <sup>(3)</sup>.

<sup>(1)</sup> J.Bagaria-N.Castells-P.Larson, ”An  $\Omega$ -logic primer”;  
H.Woodin, “The Axiom of Determinacy, Forcing Axioms, and the Nonstationary Ideal”;

<sup>(2)</sup> J.Hamkins-B.Löwe, “The Modal Logic of Forcing”;

<sup>(3)</sup> S.Leśniewski, “Collected Works”; A.Varzi, “Mereology” (Stanford Encyclopedia of Philosophy)

## 17 Peter Holy (University of Bonn)

I am working in set theory with a focus on forcing, definability and large cardinals. Some more specific topics in my recent research include class forcing and the *strength of the class forcing theorem* (that is the statement that suitable forcing relations exist for every notion of class forcing), and the use of elementary embeddings that map their critical point to a given large cardinal in order to characterize large cardinals, and applications of such *small embedding characterizations*, in particular the characterization of former large cardinals in collapse forcing extensions, making use of suitable combinatorial principles. A further topic of recent interest are certain hierarchies of *Ramsey-like cardinals*, that can equivalently be defined in

terms of elementary embeddings between certain set-sized models, or through the non-existence of winning strategies in certain infinite games.

## 18 Radek Honzik (Charles University)

*University:* Assistant professor, Charles University, Prague, Czech Republic  
*Web page:* <http://logika.ff.cuni.cz/radek>

I have been studying various compactness principles which can consistently hold at small cardinals (successors, singular cardinals). A typical example of such a property is the tree property at  $\kappa^+$  (i.e. every  $\kappa^+$ -tree has a cofinal branch), or the weak tree property at  $\kappa^+$  (i.e. there are no special  $\kappa^+$ -Aronszajn trees, which is equivalent to  $\neg \square_{\kappa}^*$ ).

Jointly with Sy-David Friedman and Šárka Stejskalová, we have recently showed that the tree property can hold at  $\aleph_{\omega+2}$ ,  $\aleph_{\omega}$  strong limit, and  $2^{\aleph_{\omega}}$  can be  $\aleph_{\omega+n}$  for any  $2 \leq n < \omega$ .

## 19 John Howe (University of Leeds)

I am a PhD student at the University of Leeds, under the supervision of John Truss and Andrew Brooke-Taylor. My interests lie at the more combinatorial end of set theory, particularly as it relates to Ramsey Theory. I am only a year into my PhD so I am not firmly tied down yet. So far I have mainly tried to look at different Ramsey properties of the reals and how they relate to each other. I have more recently been looking at the Ramsey degrees of homogeneous structures, particularly trying to examine it in the context of universal hypergraphs.

## 20 Manuel Inselmann (Universität Wien)

I am a PhD student at the Kurt Gödel Research Center in Vienna, working under the supervision of Prof. Benjamin Miller.

Over the last decades, countable Borel equivalence relations became objects of great interest in descriptive set theory (see [2],[3],[6]). Several notions of reducibility, most prominently Borel reducibility and measure reducibility, have been the focus of research. The presence of a measure often allows one to gain much more insight than one is able to get in a purely Borel context. A well-known example of this comes from the notion of the  $\mu$ -cost of an  $E$ -invariant Borel probability measure  $\mu$  ([4],[8]), which provides a plethora of theorems, including a positive answer to a weak version of a dynamic analogue of the von Neumann conjecture ([7]).

It has been known for some time that there are continuum-many pairwise incomparable countable Borel equivalence relations under Borel reducibility ([1]). Nevertheless, both under Borel and measure reducibility it remains unknown whether there exist successors of  $E_0$ .

My research efforts have been focused on generalizing the notion of cost to quasi-invariant probability measures. Another goal of this PhD project is to systematically scrutinize possible candidates that arise from actions of algebraic groups, using methods from ergodic theory and descriptive set theory.

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## 21 Lukas Daniel Klausner (Technische Universität Wien)

A cardinal characteristic is the minimal size of a set with certain properties. Many cardinal characteristics arise in quite a natural way, e. g. by considering certain ideals (such as the ideal  $\mathcal{M}$  of meagre subsets of  $\omega^\omega$  or the ideal  $\mathcal{N}$  of sets of Lebesgue measure zero). Several inequalities between such cardinal characteristics have been proven in the late twentieth century (some of which are collected in Cichoń's diagram and van Douwen's diagram). For a good overview, see Andreas Blass's "Combinatorial Cardinal Characteristics of the Continuum" in the *Handbook of Set Theory*, pp. 395–489, and Jerry E. Vaughan's "Small Uncountable Cardinals and Topology" in *Open Problems in Topology*, pp. 195–218, as well as Tomek Bartoszyński and Haim Judah's *Set Theory: On the Structure of the Real Line*. One current area of research is to construct models where more than two different cardinal characteristics which are known to be in some inequality relation are forced to be different. To this end, a number of different methods has been developed, such

as iterated forcing (with different restrictions on the support), matrix iterations, iterations along a template and creature forcing constructions.

In my thesis, which is supervised by Martin Goldstern of TU Wien (formerly also known in English as Vienna University of Technology), I aim to prove new results on the simultaneous separation of cardinal characteristics. Currently, we are working on improving creature forcing methods to separate cardinal characteristics; specifically, we are working on combining the recent results from “Creature Forcing and Five Cardinal Characteristics in Cichoń’s Diagram” (by Arthur Fischer, Martin Goldstern, Jakob Kellner, and Saharon Shelah, to appear in *Arch. Math. Logic*) with the older results from “Many Simple Cardinal Invariants” (by Martin Goldstern and Saharon Shelah, *Arch. Math. Logic*, 32(3):203–221, 1993) and plan to investigate further how much more can be achieved using products of creature forcing constructions.

Additionally, we are also interested in paradoxical consequences of non-AC in the theory of forcing.

## 22 Marlene Koelbing (University of Vienna)

I am a PhD student of Prof. Sy David Friedman at the Kurt Gödel Research Center of the University of Vienna. I am working mostly in the research area of the generalised Baire space. My main research topics are (Hausdorff) gaps, Souslin trees and Aronszajn trees, and the theory of iterated forcing, all in the context of the generalised Baire space.

For instance, I consider various generalisations of the notion of Hausdorff gaps on cardinals larger than  $\omega$  and try to develop new forcing iteration strategies to get results similar to the respective well-known results of the classical case. I also explore generalised versions of Martin’s axiom (capturing properties of models obtained by iterated forcing). Moreover, I investigate tree forcings from the abstract point of view; in particular, I work on the behaviour of Laver-type forcings on  $\kappa$ , which are quite different from classical Laver forcing.

## 23 Adam Kwela (University of Gdańsk)

My main research topics are ideals on countable sets and small subsets of the real line.

I am mostly interested in applications of ideals to topology and real analysis such as ideal convergence and descriptive complexity of ideals. These subjects are closely related to some combinatorial properties of ideals and various orders on ideals.

I am also investigating additivity and other cardinal coefficients related to certain ideals on the real line.

## 24 Marta Kwela (University of Gdańsk)

I am a PhD student at the University of Gdask. My research interests lie in the intersection of set theory, combinatorics and number theory. Currently, I am investigating ideals of nowhere dense sets in some topologies on integers. My advisor is Prof. Andrzej Nowik.

## 25 Chris Le Sueur (University of East Anglia)

I am a post-doc at the University of East Anglia, and my research is mainly in Determinacy.

It is a well-known result of Martin that ZFC proves the determinacy of all Borel games, but there has been much work in calculating how much one needs to prove the determinacy of larger and smaller pointclasses. Higher up, it is known that  $\Pi_1^1$  determinacy holds if and only if  $0^\sharp$  exists, whilst lower down the reverse mathematics of determinacy hypotheses has been largely settled, and recent work of Montalbán and Shore shows that full second order arithmetic is insufficient to prove the determinacy of  $\Delta_4^0$  games.

The method of proving determinacy in the  $\Pi_1^1$  difference hierarchy suggests a way to transfer the low-down results higher up, resulting in theorems about determinacy following from the existence of certain kinds of mice, with a fusion of methods having on the one hand a set theoretic flavour with mice, forcing and ultrapowers, and on the other a reverse mathematical flavour with subsystems of second order arithmetic. In order to push the basic method, several techniques have had to be developed, including a way of applying class forcing in weak contexts through the use of fine-structure and ramified forcing, preservation results in ultrapowers over weak models, and generalised effective descriptive set theory.

I am currently trying to establish lower bounds in the same area, using techniques similar to those invented by Welch (in the  $\Pi_1^1$  difference hierarchy) and Friedman, Martin, Montalbán and Shore (in the low Borel hierarchy.)

## 26 Marc Lischka (ETH Zürich)

**Definition 0.1.** A set  $x \in [\omega]^\omega$  is said to **bisect** a set  $y \in [\omega]^\omega$ , iff the following holds:

$$\lim_{n \rightarrow \infty} \left( \frac{|x \cap y \cap n|}{|y \cap n|} \right) = \frac{1}{2}$$

A **bisecting family** is a family  $\mathcal{S} \subseteq [\omega]^\omega$  such that each  $y \in [\omega]^\omega$  is bisected by at least one  $x \in \mathcal{S}$ .

Define the **bisecting** or **density-splitting number**  $\mathfrak{s}_{1/2}$  as follows:

$$\mathfrak{s}_{1/2} := \min\{|\mathcal{S}| : \mathcal{S} \subseteq [\omega]^\omega \text{ is bisecting}\}.$$

This is a well-defined cardinal characteristic, since  $[\omega]^\omega$  itself is an example of a bisecting family.

We can show that  $\mathfrak{s} \leq \mathfrak{s}_{1/2} \leq \mathbf{non}(\mathcal{L})$  and would like to show that either inequality can consistently be strict.

## 27 Dan Saattrup Nielsen (University of Bristol)

I am a first year PhD student at the University of Bristol, under the supervision of Philip Welch. I am currently working on so-called *Ramsey-like* cardinals, which loosely can be characterised as large cardinals between the weakly compact cardinals and measurable cardinals in terms of consistency strength that are characterised by the existence of small iterable structures. The cardinals were investigated by Gitman (2011) and Gitman & Welch (2011), and recently Holy & Schlicht (2017) have introduced a new class of Ramsey-like cardinals, the  $\alpha$ -Ramsey cardinals, which all lie strictly between Ramsey cardinals and measurable cardinals. These  $\alpha$ -Ramsey cardinals also admit a game-theoretic definition. I am also interested in the current development of descriptive inner model theory, where I am presently learning about the fine structural analysis of HOD, following Steel & Woodin (2016), and the core model induction, following Schindler & Steel (2014).

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## 28 Ana Njegomir (University of Bonn)

I am a PhD student at the University of Bonn. My supervisor is Prof. Dr. Peter Koepke. I am interested in the interplay between forcing, large cardinals and combinatorial principles.

In particular, I use forcing with finite sequences of models of two types, which was introduced by Itay Neeman, to characterize large cardinals through the validity of combinatorial principles in its forcing extensions.

That is, some principle is forced to hold if and only if the cardinal that becomes  $\aleph_2$  in the forcing extension, has the corresponding large cardinal property in the ground model. For example, assuming that  $\theta$  is regular and  $\delta < \theta$  implies  $\delta^\omega < \theta$ , together with Peter Holy and Philipp Lücke we proved that  $\theta$  is inaccessible if and only if there are no weak Kurepa trees in the generic extensions. Also, we show that assuming that  $\kappa$  is inaccessible, we have that  $\kappa$  is Mahlo if and only if there

are no special  $\aleph_2$ -Aronszajn trees in the forcing extensions. Based on results by Christoph Weiß, we have similar results for subtle and  $\lambda$ -ineffable cardinals.

### 29 Supakun Panasawatwong (University of Leeds)

### 30 Francesco Parente (University of East Anglia)

I am interested in mathematical logic, especially set theory and its interactions with model theory. Currently, I am reading for a PhD in mathematics at the University of East Anglia, under the supervision of Prof. Mirna Džamonja and Dr David Asperó; I expect to complete my degree in October 2018. Before starting my PhD, I received my Master's degree from the University of Pisa, with a thesis on *Boolean-valued models, saturation, forcing axioms* supervised by Prof. Matteo Viale.

Over the last decade, Malliaris and Shelah proved a striking sequence of results in the intersection between model theory and set theory, settled affirmatively the question of whether  $\mathfrak{p} = \mathfrak{t}$ , and developed surprising connections between classification theory and cardinal characteristics of the continuum. The main motivation of their work is the study of Keisler's order, introduced originally in 1967 as a device to compare the complexity of first-order theories by looking at saturated ultrapowers of their models.

My ongoing research project investigates the set-theoretic aspects of Keisler's order, with a particular focus on the the Boolean ultrapower construction. Indeed, although the definition of Keisler's order makes use of regular ultrafilters on power-set Boolean algebras, recently there has been a shift towards building ultrafilters on complete Boolean algebras. In particular, moral ultrafilters have emerged as the main tool to find dividing lines among unstable theories.

Motivated by this new Boolean-algebraic framework, the main questions addressed in my research are the following: what kind of classification can arise when we compare theories according to the saturation of Boolean ultrapowers of their models? How does the combinatorial structure of Boolean algebras relate to the complexity of first-order theories? The techniques involved are the construction of regular ultrafilters using large cardinals and the investigation of model-theoretic properties of good and OK ultrafilters on complete Boolean algebras.

### 31 Robert Passmann (University of Amsterdam)

I am a student of the Master of Logic at the University of Amsterdam where I specialise in the track "Logic and Mathematics". Previously, I completed a bachelor's degree in mathematics at the University of Bonn where I focused on set theory.

Right now, my main interests in set theory are connections between (abstract) logics and large cardinals. It is possible to relate set-theoretic principles to model-theoretic principles (such as Löwenheim-Skolem-Theorems). With a group of people in Amsterdam, we are trying to find set-theoretic principles that relate to compactness theorems.

Moreover, I am interested in forcing techniques, (generalised) descriptive set theory and multiverses in set theory. I enjoy discussions about the philosophy of set theory and/or mathematics.

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## 32 Márk Póór (Eötvös University)

I am a first year Phd student at Eötvös University, Budapest, working under the supervision of Márton Elekes. My main research interests are set theory of the reals, cardinal invariants of the continuum and descriptive set theory.

The notion of Haar-null sets (in the sense of Christensen’s [2]) in Polish groups is the generalisation of the null ideal w.r.t. the Haar measure in (Polish) locally compact groups, which has been very useful in studying exceptional sets in diverse areas of mathematics. I will mention some interesting problems concerning this Borel ideal.

It is known that  $\text{add}(\mathcal{HN}) = \omega_1$  [3], and in joint work with Márton Elekes we proved that  $\text{cof}(\mathcal{HN}) = c$  in groups admitting an invariant metric. However the covering number and the ”non” of the Haar-null sets admitting universally measurable hulls are determined (in Polish groups of some special forms) [1], we still don’t know much about  $\text{cov}(\mathcal{HN})$ ,  $\text{non}(\mathcal{HN})$  (for the Borel ideal).

There exists null but non-meager subgroups in  $\mathbb{R}$  [7], and also in every non-discrete locally compact group [4], whereas it is consistent that there is no null non-meager subgroup in each non-discrete locally compact group [4]. The question of existence of such subgroups are open for non-locally compact Polish groups.

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### 33 Alejandro Poveda (Universitat de Barcelona)

I am an officially-enrolled doctoral student in the University of Barcelona doctoral program on Pure and Applied Logic and I holding a FPU 4-years doctoral fellowship awarded by the Spanish Government. Nowadays I am working under the supervision of Professor J. Bagaria.

My personal research can be framed in the interplay between the technique of Forcing and Large Cardinal Axioms. More precisely, I am strongly interested in the study of Generic Absoluteness Principles as Woodin's Absoluteness (and its recent generalization due to M. Viale), Resurrection Axioms and the study of the universe in the region encompassed between Strongly compact cardinals and Supercompact using Prikry-type forcings.

### 34 Vibeke Quorning (University of Copenhagen)

I am a PhD student within descriptive set theory at the University of Copenhagen. My advisor is Asger Törnquist. Currently, I am studying co-analytic ranks on countable Polish metric spaces.

My other research interests includes countable group actions and their induced orbit equivalence relations from both the ergodic theoretic and the descriptive set theoretic point of view. Within these areas I am in particular interested in Borel reducibility among profinite actions and treeable equivalence relations.

### 35 Millette Riis (University of Leeds)

I am a second year PhD student at the University of Leeds under supervision of Professor John Truss.

My research focuses on colourings of the infinite generalised shift graph. In 1968, Erdős defined the Shift Graph as the graph whose vertices are the  $k$ -element subsets of  $[n] = \{0, 1, 2, \dots, n-1\}$  such that  $A = \{a_1, \dots, a_k\}$  and  $B = \{b_1, \dots, b_k\}$  are neighbours iff  $a_1 < b_1 = a_2 < b_2 = a_3 < \dots < b_{n-1} = a_n < b_n$ . In the paper *On the Generalised Shift Graph*, Avart, Luczac and Rödl extend this definition to include all possible arrangements of the  $a_i$ s and  $b_i$ s, known as *types*. I have considered a selection of these types and studied their corresponding graphs.

I am interested in to what extent the graphs  $G(S, \tau)$  and  $G(S', \tau)$  are distinct for distinct linear orderings  $S, S'$  and for some type  $\tau$ . I have concentrated on ordinals and types of the form  $\sigma_{a,b} = 11\dots 133\dots 322\dots 2$ , and shown that if  $G(\alpha, \sigma_{a,b}) \cong G(\beta, \sigma_{a,b})$  then  $\alpha = \beta$ . I have also considered the chromatic number and the automorphism groups of these graphs in order to gain a deeper understanding of their properties.

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### 36 Noah Schoem (University of Illinois at Chicago)

I am a third year Ph.D student, working under Dima Sinapova, primarily interested in using inner model theory and forcing to determine large cardinal equiconsistencies.

Forcing often gives one direction of equiconsistency results; such as using Cohen forcing to obtain  $Con(ZFC) \implies Con(ZFC + \neg\text{Continuum Hypothesis})$ , or to show that one may force, from a weakly compact, the tree property at  $\aleph_2$ , or from a supercompact cardinal, the consistency of the Proper Forcing Axiom. [1]

Inner Model Theory often proves reverse directions of equiconsistency results. There is a canonical inner model  $L$  that is used in some equiconsistency proofs; for example, if  $V \models TP_{\aleph_2}$  (where  $TP_\kappa$  is the tree property at  $\kappa$ ), then  $L \models \text{“}\aleph_2 \text{ is weakly compact”}$ ; hence  $Con(ZFC+TP_{\aleph_2}) \implies Con(ZFC+\exists\kappa \text{ weakly compact})$ . [1]

Due to Jensen’s Covering Lemma, if  $0^\#$  exists then  $V \neq L$ ; so for large cardinal consistency results involving measurable cardinals and larger, larger inner models are necessary, such as  $L[U]$ , where  $U$  is a normal measure on some  $\kappa$ , or  $L[\mathcal{U}]$  where  $\mathcal{U}$  is an extender. [1] These models all fairly  $L$ -like, in that they exhibit a fine structure theory and uniformly satisfy some square principle. There is no such inner model for a supercompact cardinal, and some reasons to believe that there isn’t a good candidate for one. [2]

It is known that from a supercompact, one can obtain the consistency of the Proper Forcing Axiom. Whether the converse holds is still open, although partial results are known. Given an outer model  $W = V[\mathbb{P}]$  where  $W \models PFA$  and certain classes of forcings  $\mathbb{P}$ ,  $V \models \text{“}\omega_2^W \text{ strongly compact”}$ ; when  $\mathbb{P}$  is proper,  $V \models \text{“}\omega_2^W \text{ supercompact”}$ . [3] I am interested in further work on calibrating the consistency strength of  $PFA$  and any inner model theory concerning supercompacts.

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### 37 Salome Schumacher (ETH Zurich)

I am a PhD student at ETH Zurich working under the supervision of Lorenz Halbeisen. At the moment I am mainly interested in magic sets and weak choice principles.

A set  $M \subseteq \mathbb{R}$  is a magic set if for all nowhere constant and continuous functions  $f$  and  $g$

$$g[M] \not\subseteq f[M] \iff f \neq g.$$

I showed that adding and removing countable sets does not destroy the property of being magic. If  $\text{add}(\mathcal{M}) = \mathfrak{c}$ , we can even add and remove sets of cardinality less than  $\mathfrak{c}$ . An overview about this topic can be found on my poster.

$C_n^-$  states that every infinite family  $\mathcal{F}$  of sets of size  $n$  has an infinite subset  $\mathcal{G} \subseteq \mathcal{F}$  with a choice function on  $\mathcal{G}$ . And  $RC_n$  states that every infinite set  $X$  has an infinite subset  $Y \subseteq X$  such that  $[Y]^n = \{z \subseteq Y \mid |z| = n\}$  has a choice function. I am interested in the question for which  $n \in \omega$  the implication  $RC_n \Rightarrow C_n^-$  holds.

### 38 Isabella Scott (University of St Andrews)

I just completed my undergraduate and will be starting a graduate programme at the University of Chicago in the fall. My undergraduate dissertation was on applications of forcing and focussed on Todorcevic's work on forcing and compact subsets of the first Baire class. However, I hope to use this conference to explore the breadth set theory and find topics I would be interested in pursuing in a PhD.

### 39 Forte Shinko (McGill University)

My main interest is understanding the Borel complexity of orbit equivalence relations arising in geometric group theory. For the past two years at McGill University, I have been working with Jingyin Huang and my supervisor Marcin Sabok, investigating actions of hyperbolic groups and their induced actions on the Gromov boundary. Since the Gromov boundary is a Polish space, it is natural to talk about the Borel bireducibility class of the induced orbit equivalence relation on the boundary; in particular, we have shown that every hyperbolic group acting properly and cocompactly on a CAT(0) cube complex induces a hyperfinite Borel equivalence relation on the Gromov boundary. This generalizes the well-known result of

Dougherty-Jackson-Kechris stating that the tail equivalence relation is hyperfinite. There is still a fair bit of work to be done in this direction, looking at more general actions of hyperbolic groups, or actions of mapping class groups and other similar families.

#### 40 **Damian Sobota** (Universität Wien)

My main research interests circulate around applications of set theory to Banach space theory and measure theory. In particular, I am interested in relations between cardinal characteristics of the continuum and issues of convergence of sequences of measures on Boolean algebras, equivalently on totally disconnected compact spaces. Currently, I study preservation of measure-theoretic properties of Boolean algebras (such as the Nikodym property or Grothendieck property) in forcing extensions.

#### 41 **Šárka Stejskalová** (Charles University)

*University:* Ph.D. student, Charles University, Prague, Czech Republic

*Web page:* <http://logika.ff.cuni.cz/sarka>

In my Ph.D. studies (which I plan to finish this year) I have focused on the tree property at  $\kappa^+$  (every  $\kappa^+$ -tree has a cofinal branch), and the weak tree property (there is no  $\kappa^+$ -Aronszajn tree), and the interplay with the continuum function. As is well known the tree property at  $\kappa^+$  implies  $\kappa^{<\kappa} > \kappa$ , and therefore for instance the tree property at  $\aleph_2$  decides the continuum hypothesis negatively.

Jointly with Radek Honzik, I have shown that the tree property does not put any further restrictions on the continuum function below  $\aleph_\omega$  (strong limit) in the sense that the tree property can hold at each  $\aleph_{2n}$ ,  $0 < n < \omega$ , and the continuum function can be any non-decreasing function taking values below  $\aleph_\omega$  which satisfies  $2^{\aleph_{2n}} \geq \aleph_{2n+2}$ ,  $0 \leq n < \omega$ . A similar result holds for the weak tree property holding at each  $\aleph_n$ ,  $1 < n < \omega$ .

Recently, I have studied (jointly with Sy-David Friedman and Radek Honzik) the tree property at the double successors of singular cardinals with countable cofinality (e.g.  $\aleph_\omega$ ).

Please see my web page for preprints.

#### 42 **Jarosaw Swaczyna** (Łódź University of Technology)

#### 43 **Fabio Elio Tonti** (Vienna University of Technology)

I am a third year PhD student at the Vienna University of Technology, working under the supervision of Prof. Jakob Kellner Prof. Asger Törnquist.

In recent years, the concept of Borel reducibility among Borel equivalence relations has seen a large number of applications to different fields of mathematics, in particular classification problems in the context of  $C^*$ -algebras (see e.g. [5] or [1]) or the classification problem for torsion-free abelian groups of finite rank (see [6] and [7]). It is known that the relation of unitary equivalence on the space of

irreducible unitary representations of a discrete countable group on a fixed separable infinite-dimensional Hilbert space is a Borel equivalence relation (this holds true even for the space of all representations; see [3]). I am particularly concerned with the problem of finding a way to distinguish the complexity for the relation of unitary equivalence among different groups. From the work of E. Thoma and subsequent work by G. Hjorth it is known that if a group is abelian-by-finite, then the associate equivalence relation is smooth; else it is not even classifiable by countable structures. For a current state of affairs see [8].

Furthermore, I am interested in cardinal characteristics. The consistency of many inequalities between cardinal characteristics of the continuum is known, but often only for  $\mathfrak{c} = \aleph_2$ . The obvious question to ask here is whether these inequalities are still consistent with large continuum; or, whether three or more cardinal characteristics can be pairwise different. In order to achieve this, I am interested in further developing the creature forcing technique from [2] (see also [4]).

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### 44 Andrea Vaccaro (University of Pisa and York University)

I am a second year PhD student at University of Pisa and York University (Toronto), working under the supervision of Prof. Ilijas Farah.

My main interests are logic and applications of logic to operator algebras, and much of my work focuses on some classic problems coming from the theory of  $C^*$ -algebras which were recently studied with techniques and tools coming from set theory.

One of these is Anderson's Conjecture, which states that every pure state on  $B(H)$ , the  $C^*$ -algebra of linear bounded operators on a separable Hilbert space, can be restricted to a multiplicative state on some atomic maximal abelian subalgebra of  $B(H)$ . The conjecture was shown to be consistently false, and this work led to the introduction and study of some filter-like sets (known as quantum filters), generalizing in the non-commutative framework the classic notion of ultrafilter ([1]). The question whether Anderson's Conjecture is consistent is wide open.

Another main topic of my research is Naimark's Problem. The problem asks whether every  $C^*$ -algebra whose irreducible representations are all equivalent up to unitary equivalence is isomorphic to the algebra of the compact operators. Assuming Jensen's diamond, in [2] Akemann and Weaver were able to produce  $C^*$ -algebras witnessing that the answer to the above question is (consistently) no. Recently I have been studying these algebras and the Akemann and Weaver' construction, in order to understand how complicated and versatile their K-theory and trace space can be.

I am also interested in the non-abelian generalization of Stone-Weierstrass Theorem ([3]). Unlike the two problems discussed above, no consistency result is known for this one. By means of techniques coming from functional analysis some partial results were obtained in the past, but it would be interesting to know if tools coming from model theory or set theory could give new insights and results about this problem.

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**45 Luis Valencia** (University of Barcelona)

**46 Jonathan L. Verner** (Charles University in Prague)

Most of my research interests center around filters and ultrafilters, typically on  $\omega$ . Recently I've been looking at the structure of the Rudin-Keisler order. For example, in a joint work with Dilip Raghavan we obtained the following theorem:

**Theorem:** Assume CH. The RK-order of rapid P-points is  $\mathfrak{c}^+$ -closed (upwards).

A different topic I am interested in is the interplay between ultrafilters and forcing. There is a standard theorem which allows one to preserve P-points in some forcing extensions. Not much is known about other ultrafilters though. Only quite recently S. Shelah came with a complicated construction of an ultrafilter which is not a P-point (nor gotten out of a P-point) which can be preserved.

## 47 Louis Vuilleumier (Université de Lausanne, Switzerland)

Supervisor: *Jacques Duparc*

The title of my research is *Continuous Reductions on Quasi-Polish Spaces*. It lies between the more general areas of Descriptive Set Theory and Domain Theory. Quasi-Polish Spaces were introduced by De Brecht in [1] as a generalization of both Polish spaces and  $\omega$ -continuous domains. The former being the central object of study of classical Descriptive Set Theory [2], and the latter being of great importance in computer science, in particular as models of  $\lambda$ -calculus [3].

The goal of the project is to better understand the shape of the quasi-order induced by continuous reductions on the subsets of quasi-Polish spaces. Some general results have been obtained by Motto Ros, Selivanov and Schlicht in [4] for classes of functions more general than the class of continuous functions. Specific results have been obtained by Selivanov in [5] and then by Becher and Grigorieff in [6] for the Scott domain, a universal quasi-Polish space.

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- [2] A. Kechris, *Classical Descriptive Set Theory*, *Graduate Texts in Mathematics* 156 (1995), Springer-Verlag.
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## 48 Philip Welch (University of Bristol)

My research has mostly centred on definable inner models, principally core models and their fine structure. Rather than working in forcing to discover relative consistency, I have been more motivated by discovering what is, or is not, true in  $V$ .

This has often involved investigating the interaction via covering theorems between combinatorial properties of inner models and  $V$ . Just to give an example, if there is no inner model with a strong cardinal, then if  $\kappa$  is a regular Jonsson cardinal, then not only does  $\square_\kappa$  hold in  $V$ , but so does  $\square_\alpha$  for stationary many  $\alpha < \kappa$  ([1]). Further examples are provided by looking at variants of Chang's property at  $\aleph_2$  ([2]), and the mutual stationarity property of Foreman and Magidor.

I have been interested in interactions between inner models and determinacy, often at the lower levels. An example is calculating in terms of inner models the exact strength of  $\omega^2$ - $Pi_1^1$ -Determinacy, the last case left over from the work of Martin & Harrington from the 70's. More recently I have been looking at levels of determinacy provable in analysis and locating their strategies in  $L$ . Surprisingly it turns out there is an application of infinite time Turing machines considered as an extension of Kleene's higher type recursion from the '60's, to characterize precisely  $\Sigma_3^0$ -Determinacy in terms of a generalized Kleene-esque 'halting problem'. This I regard as interesting as it gives for the first time an application of this machine model to classical descriptive set theory.

More recently still I have been involved in discussions on foundational issues, providing arguments (a "Global Reflection Principle" - GRP) that Reflection Principles can be purportedly extended to show that there is a proper class of measurable Woodin cardinals in  $V$  ([3]). A different branch of this philosophical turn is a number of papers engaging with philosophers of truth on the logical/set theoretical aspects of their assumptions usually in terms of constructibility theory. For example the paper [4] shows that Field's claim that a theory of truth gave a "revenge free solution to the semantic paradoxes" via hierarchies of determinateness operators could not work.

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[3] "Reflecting on Absolute Infinity" with L. Horsten, in Journal of Philosophy, 113, Feb. 2016, 89-111.

[4] "Some observations on Truth Hierarchies" in the Review of Symbolic Logic, 7 No. 1, March 2014 1-30.

## 49 Wolfgang Wohofsky (University of Hamburg)

I got my PhD at the Vienna University of Technology, with Martin Goldstern as my advisor. Currently, I am a Postdoc at the University of Hamburg, working in

the Mathematical Logic Group of Prof. Benedikt Löwe.

My research area is (iterated) forcing and (its applications to) set theory of the reals.

In particular, I study questions about small (or: “special”) sets of real numbers such as strong measure zero sets (i.e., by the Galvin-Mycielski-Solovay theorem, meager-shiftable sets) or other sets which can be defined in terms of translations, as well as (dual) Borel Conjecture and other variants of the Borel Conjecture (joint with Jörg Brendle we showed in ZFC that there is no set of reals of size continuum which can be translated away from every Marczewski null set).

Moreover, I work on cofinalities of ideals (together with Jörg Brendle and Yurii Khomskii we proved that the Marczewski-like ideals related to Laver or Miller forcing have cofinality strictly above the continuum in ZFC) and on cardinal characteristics of the continuum (which are in or related to Cichoń’s diagram).

I am also interested in generalizing classical concepts or questions concerning real numbers to the generalized Cantor or Baire space. For instance, I determined to which extent the Galvin-Mycielski-Solovay theorem can be generalized to uncountable cardinals. In a joint project with Yurii Khomskii, Marlene Koelbing, and Giorgio Laguzzi we examine properties of Laver-type forcings on  $\kappa$ .

# New Directions in the Higher Infinite

## Timetable

|             | <b>Monday</b>                 |             | <b>Tuesday</b>     |             | <b>Wednesday</b>  |             | <b>Thursday</b>    |             | <b>Friday</b>   |
|-------------|-------------------------------|-------------|--------------------|-------------|-------------------|-------------|--------------------|-------------|-----------------|
| 9.00-10.00  | Registration                  | 9.00-10.00  | Joan Bagaria       | 9.00-10.00  | Grigor Sargsyan   | 9.00-10.00  | Asger Törnquist    | 9.00-10.00  | Asger Törnquist |
| 10.00-11.00 | Joan Bagaria                  | 10.00-11.00 | Joan Bagaria       | 10.00-11.00 | Grigor Sargsyan   | 10.00-11.00 | Asger Törnquist    | 10.00-11.00 | Asger Törnquist |
| 11.00-11.30 | Coffee break                  | 11.00-11.30 | Coffee break       | 11.00-11.30 | Coffee break      | 11.00-11.30 | Coffee Break       | 11.00-11.30 | Coffee break    |
| 11.30-12.30 | Joan Bagaria                  | 11.30-12.30 | Carolin Antos      | 11.30-12.30 | David Aspero      | 11.30-12.30 | Thilo Weinert      | 11.30-12.30 | Silvia Steila   |
| 12.30-14.00 | Lunch break                   | 12.30-14.00 | Lunch break        | 12.30-14.00 | Trevor Wilson     | 12.30-14.00 | Lunch break        | 12.30-14.00 | End             |
| 14.00-15.00 | Mirna Džamonja                | 14.00-15.30 | Discussion session | 14.00-15.30 |                   | 14.00-15.30 | Discussion session | 14.00-15.30 |                 |
| 15.00-16.00 | Matthew Foreman               | 15.30-16.00 | Coffee break       | 15.30-16.00 |                   | 15.30-16.00 | Coffee break       | 15.30-16.00 |                 |
| 16.00-16.30 | Coffee break                  | 16.00-17.00 | Matthew Foreman    | 16.00-17.00 |                   | 16.00-17.00 | Grigor Sargsyan    | 16.00-17.00 |                 |
| 16.30-17.30 | Matthew Foreman               | 17.00-18.00 | Matthew Foreman    | 17.00-18.00 |                   | 17.00-18.00 | Grigor Sargsyan    | 17.00-18.00 |                 |
| 17.30-19.00 | Poster session/Wine reception |             |                    |             | Conference dinner |             |                    |             |                 |