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Noncommutative thin-tall algebras

Piotr Koszmider

Institute of Mathematics of the Polish Academy of Sciences, Warsaw

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A Boolean ring is called thin-tall if it is generated by a tower of partitions of ω of length ω_1.

How to make a tower of partitions of ω of length ω_1 in ZFC

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- is essential in $C^*(\mathcal{P} \cup \mathcal{Q})$, i.e., for any nonzero element $a \in C^*(\mathcal{P} \cup \mathcal{Q})$ there is $b \in C^*(\mathcal{Q})$ such that $ab \neq 0$.

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We say that a system of matrix units $\mathcal{P}$ is coarser than a system of matrix units $\mathcal{Q}$ whenever the C*-algebra $C^*(\mathcal{Q})$ generated by $\mathcal{Q}$

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Fully noncommutative thin-tall C*-algebras

Definition

$\mathcal{P}$ is a system of matrix units in $\ell_2(\mathbb{N})$ if and only if

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Proof.

Build a tower of partition which has a refinement which is a Luzin almost disjoint family (where no two disjoint uncountable subfamilies can be separated). □

Automorphisms of thin-tall algebras

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