Resolvability of topological spaces

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Set Theory, Model Theory and Applications,

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Thus, if X is $\mathcal{D}$-forced and we know $\mathcal{D}$ then we also know all the dense subsets of X.

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