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Title: Games on weakly compact cardinals

Abstract: Weakly compact cardinals are equivalent to the statement that every κ -complete filter on a Boolean algebra \mathcal{B} of size κ can be extended to a κ -complete ultrafilter on \mathcal{B} . One can continue this finitely many times. Can it be continued transfinitely?

Fix a cardinal κ and consider the following game \mathcal{G}_γ of ordinal length γ : Player I plays a sequence of collections $\mathcal{S}_\alpha \subseteq P(\kappa)$ of size κ and player II plays an increasing sequence of κ -complete ultrafilters U_α on $\bigcup_{\beta \leq \alpha} \mathcal{S}_\beta$. Player II wins if she can continue playing until stage γ .

Clearly if κ is measurable then II wins the game of any length. Welch asked whether the property that “II has a winning strategy in \mathcal{G}_γ ” can be intermediate in strength between a weakly compact and a measurable cardinal.

The main result in this talk is that if II wins \mathcal{G}_{ω_1} then there is a precipitous ideal on κ whose quotient has a countably closed dense subset. Hence the answer to Welch’s question, at least for $\gamma \geq \omega_1$, is *no*.

In joint work with Magidor, we prove that it is consistent at a non-measurable cardinal for II to have a winning strategy in \mathcal{G}_{ω_1} , hence the theorem is not vacuous.